

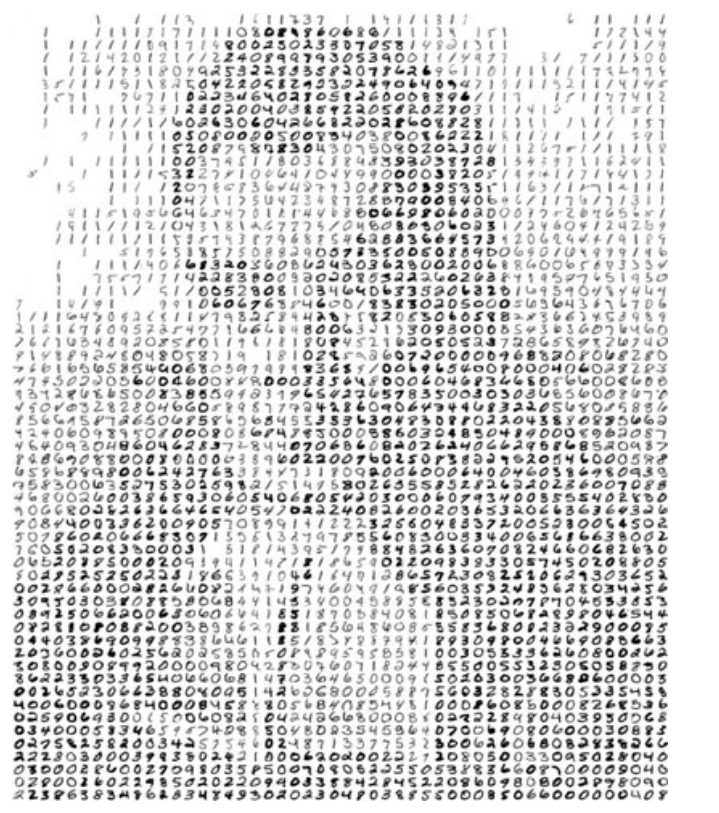


# Exploring the Influence of Particle Filter Parameters on Order Effects in Causal Learning

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## Overview

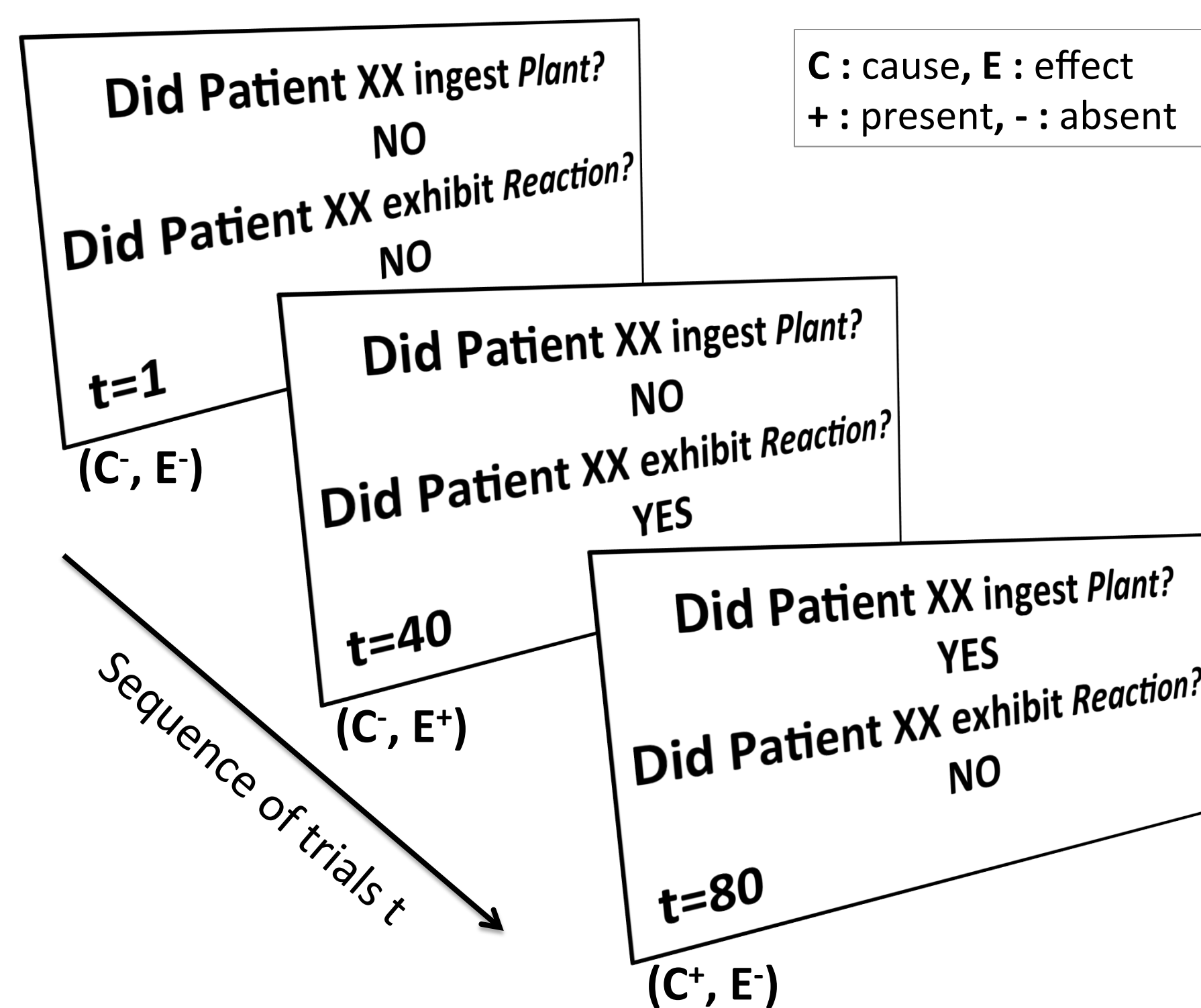
**Question:** The order in which people observe data has an effect on their subsequent judgments and inferences. How do we model this phenomenon?

**What we know:** Most Bayesian models of human behavior do not produce these effects. However, approximation methods for Bayesian inference have been shown to predict certain order effects.

**What we don't know:** How do the parameters of these approximation methods influence predictions of order effects?

**Our contribution:** We investigate the role of certain parameters in a sequential Monte Carlo method known as a *particle filter*. In a simple causal learning task, we find a particular parameter setting is responsible for producing different order effects.

## Order Effects in Causal Learning



To what extent does *Plant* cause *Reaction*?

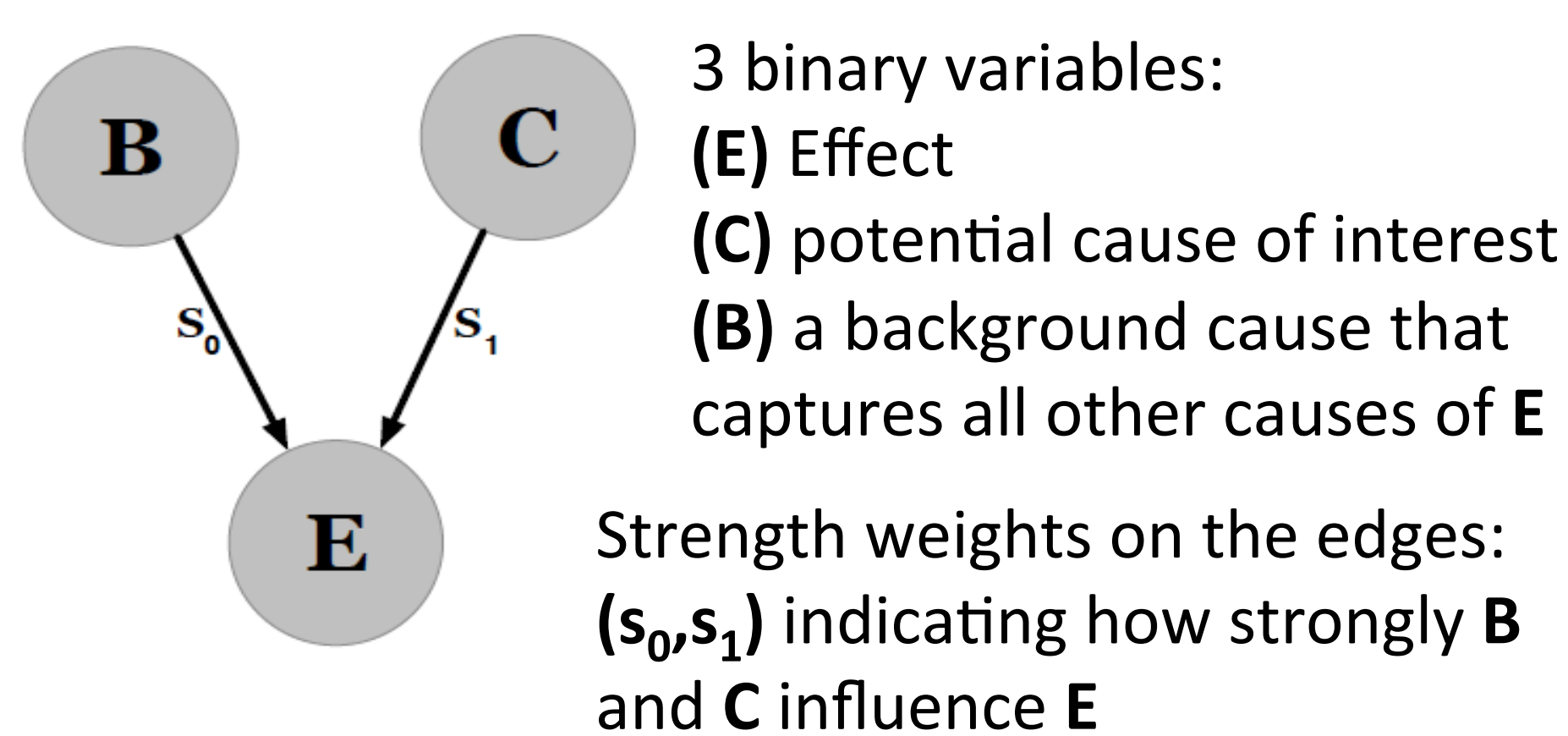
Stimuli distribution:

	Block 1 (Generative)			Block 2 (Preventative)	
	E <sup>+</sup>	E <sup>-</sup>		E <sup>+</sup>	E <sup>-</sup>
C <sup>+</sup>	18	2	C <sup>+</sup>	2	18
C <sup>-</sup>	2	18	C <sup>-</sup>	18	2

**Primacy effects:** initial information has greatest impact on later judgments. Produced when judgment question asked only at the end of the trial sequence. (Dennis and Ahn, 2001)

**Recency effects:** most recent information has greatest impact on later judgments. Produced when judgment question asked after every 10 trials. (Collins and Shanks, 2002)

## Bayesian model of Causal Learning



What is the probability of the observed data given the strength weights?

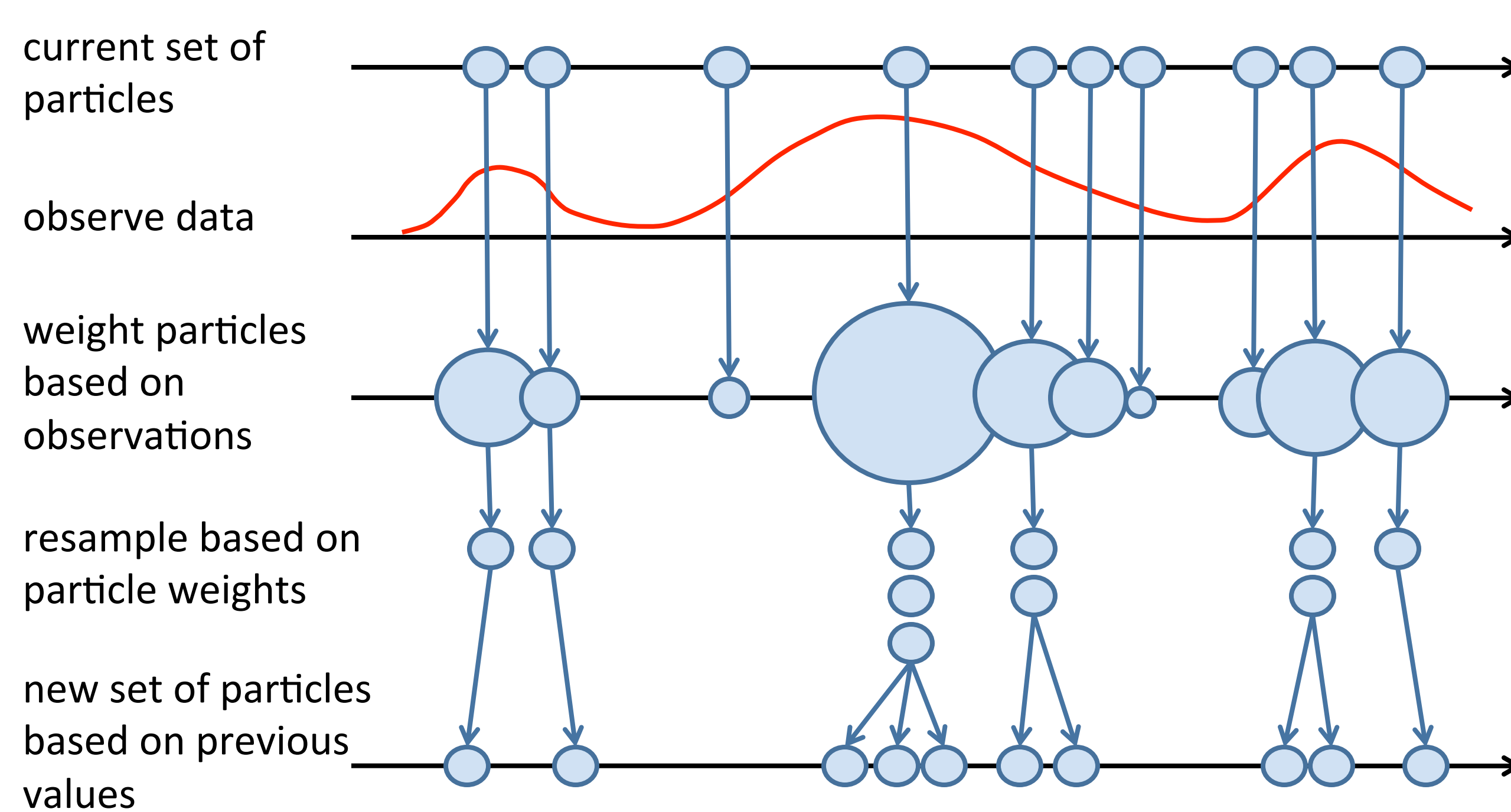
C	E	$s_1 \geq 0$	$s_1 < 0$
1	1	$s_0 + s_1 - s_0 s_1$	$s_0(1 + s_1)$
1	0	$1 - (s_0 + s_1 - s_0 s_1)$	$1 - [s_0(1 + s_1)]$
0	1	$s_0$	$s_0$
0	0	$1 - s_0$	$1 - s_0$

## Particle Filters

Assume we have a sequence of unobserved latent variables  $z_1, \dots, z_t$  where  $z_{0:t}$  is modeled as a Markov process and each  $z$  holds a pair of strength estimates  $s_0$  and  $s_1$ . Additionally, we have a sequence of observations  $y_1, \dots, y_t$  representing the covarying events. The posterior distribution  $P(z_{0:t} | y_{1:t})$  can be obtained recursively as:

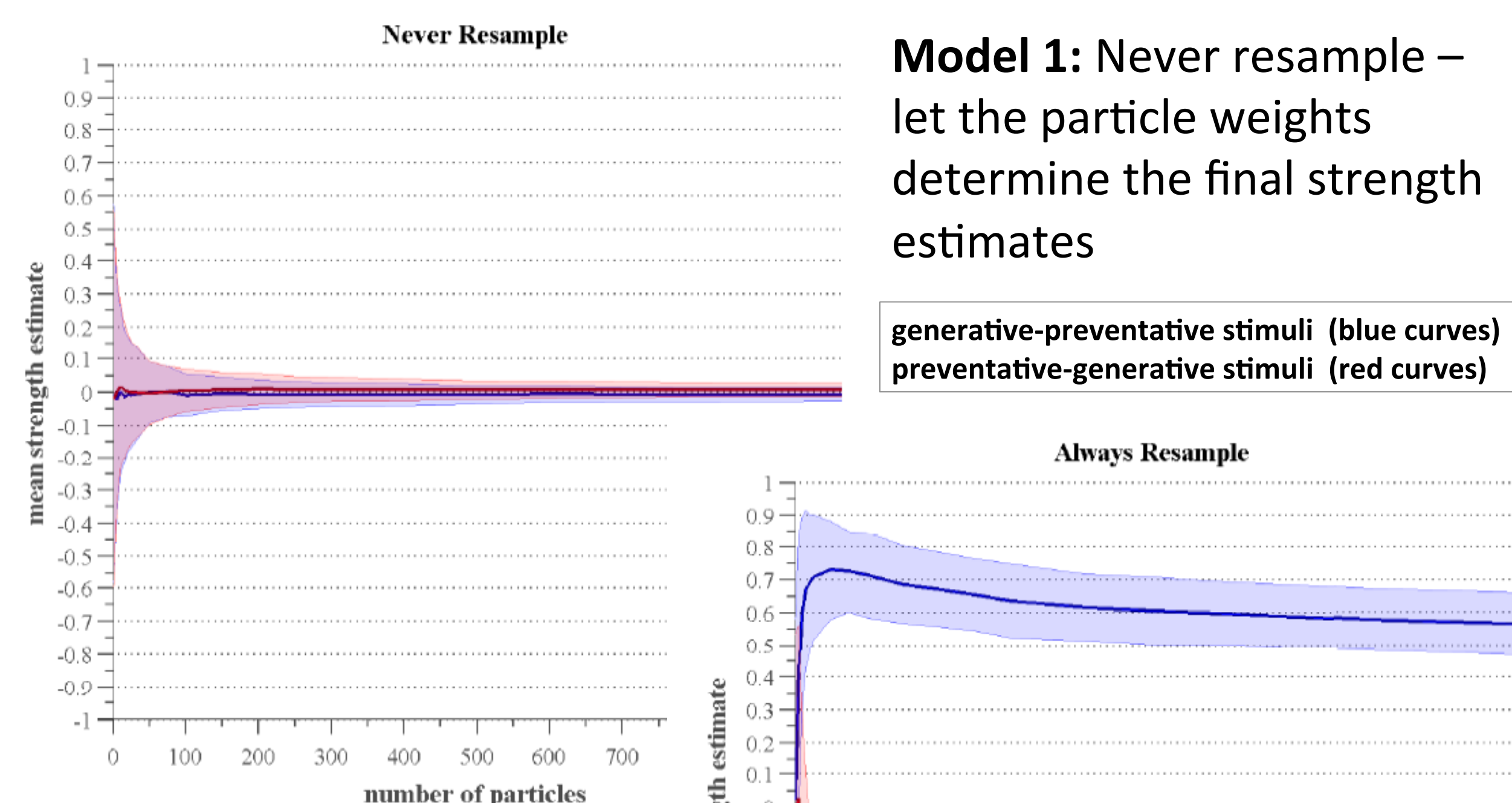
$$P(z_{0:t+1} | y_{1:t+1}) \propto P(z_{0:t} | y_{1:t}) P(y_{t+1} | z_{t+1}) P(z_{t+1} | z_t)$$

Importance sampling can be used recursively to approximate this distribution by sampling from  $P(z_{t+1} | z_t)$  for each value of  $z_t$ , weighting each value of  $z_{t+1}$  by  $P(y_{t+1} | z_{t+1})$ , and then resampling from this weighted distribution. This algorithm, in which a set of samples is constantly updated to reflect the information provided by each observation, is known as a particle filter. The samples are referred to as particles.



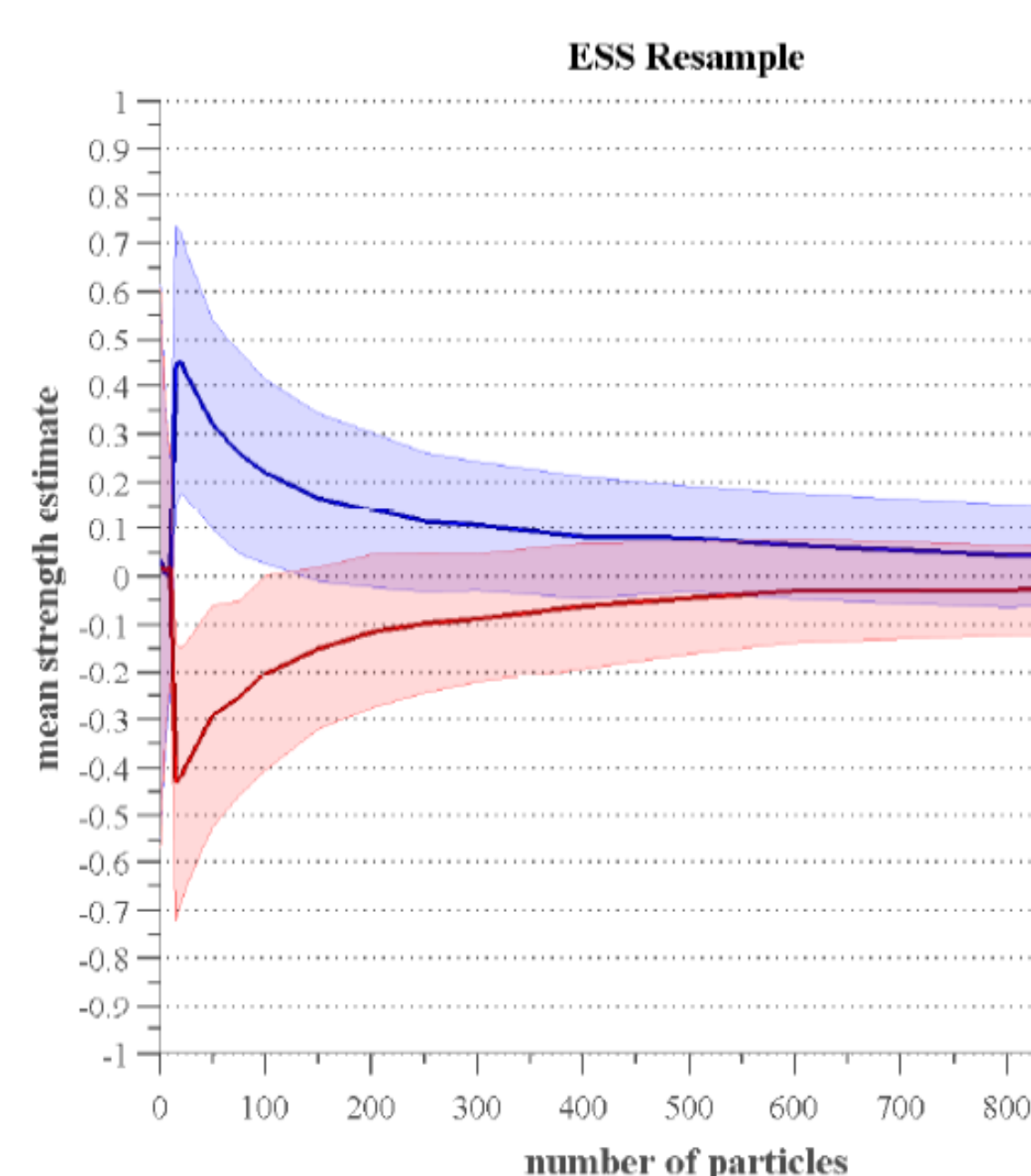
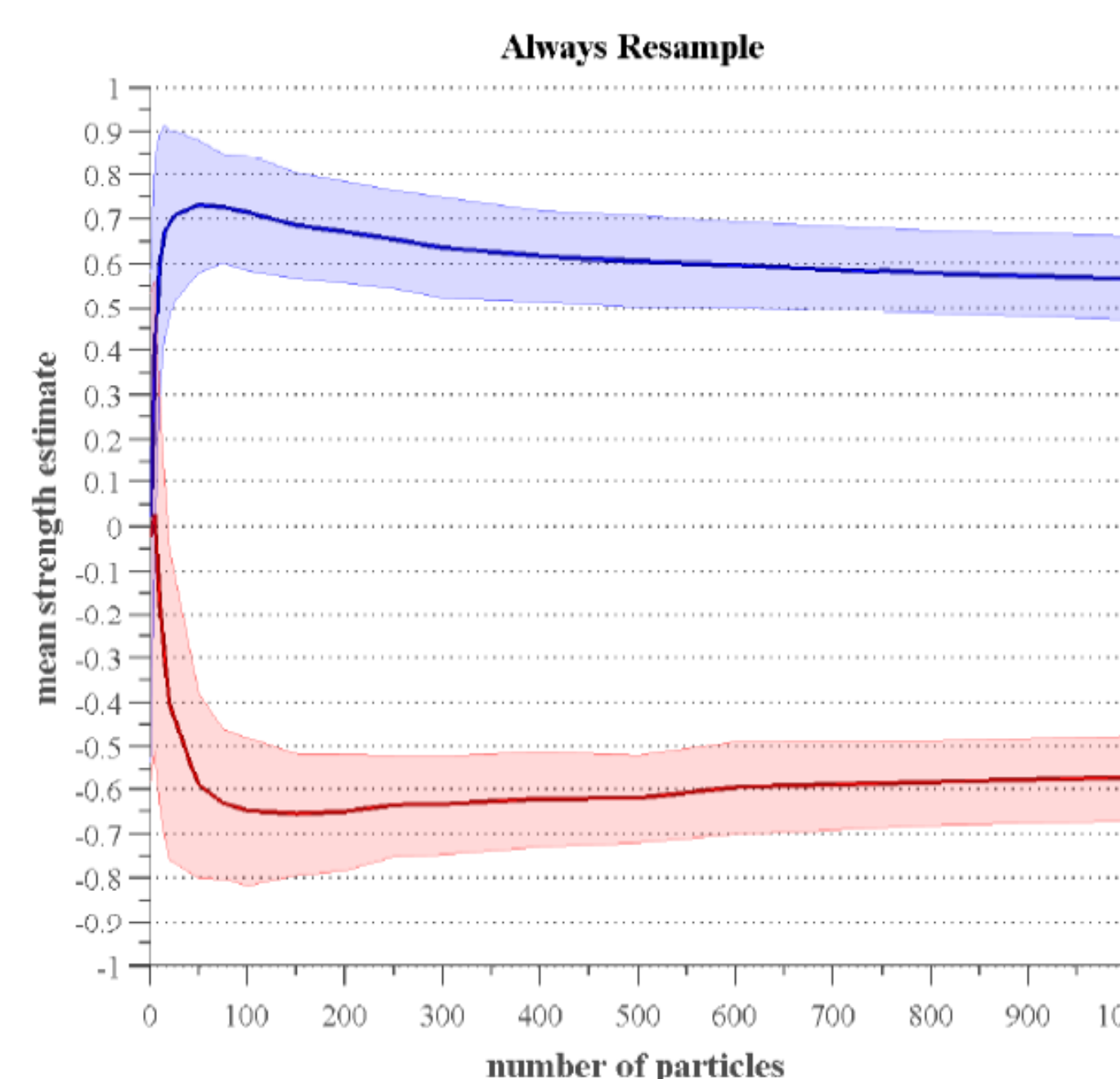
## Order Effects and Particle Filter Parameters

How do different methods of resampling influence order effects?

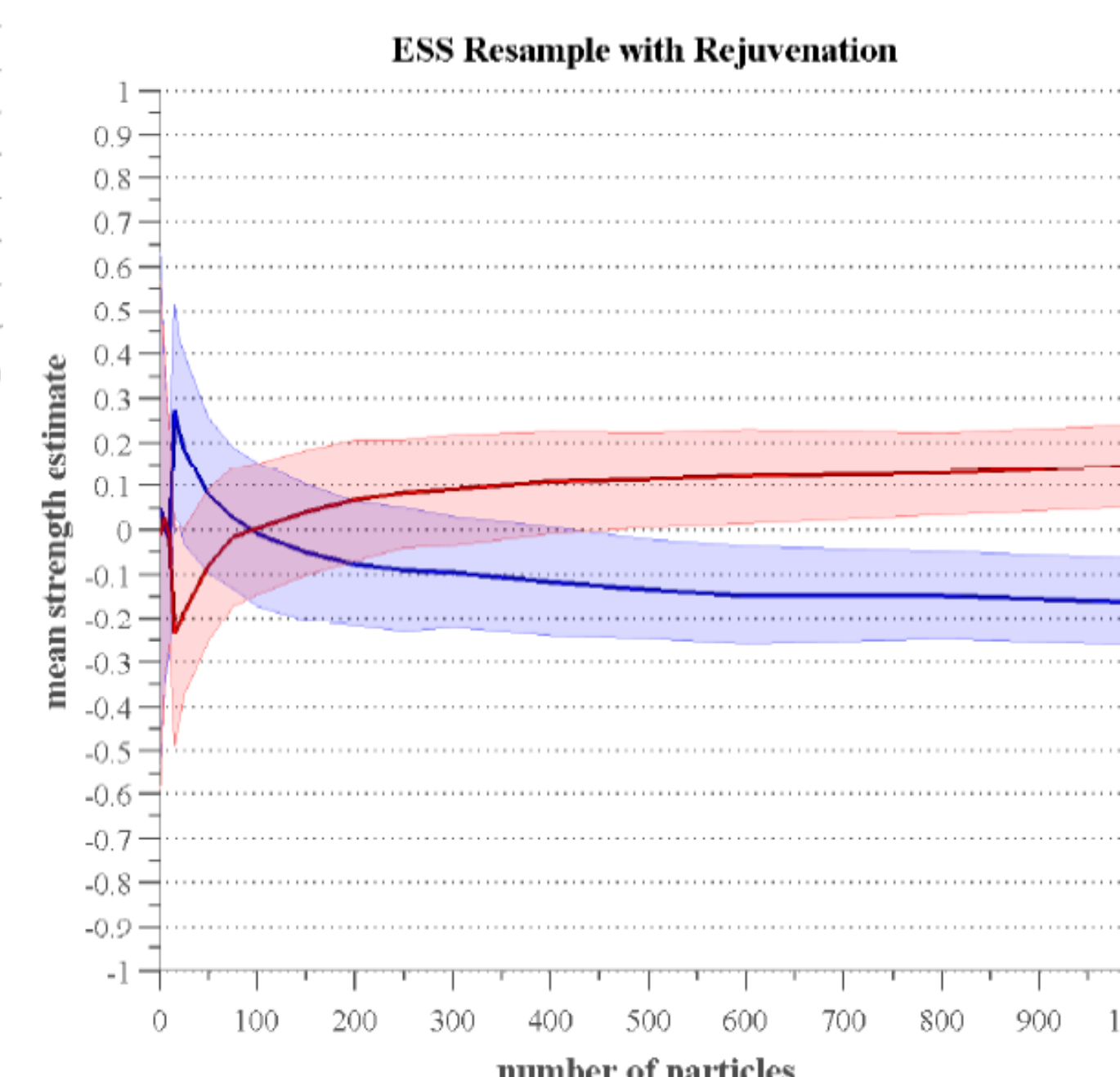


**Model 1:** Never resample – let the particle weights determine the final strength estimates

**Model 2:** Always resample – based on a multinomial distribution defined on the particle weights and at every trial  $t$



**Model 3:** ESS resample – based on a multinomial distribution defined on the particle weights and only if the variance of the weights is too large as defined by the Effective Sample Size:  $ESS = |w_t|^{-2}$

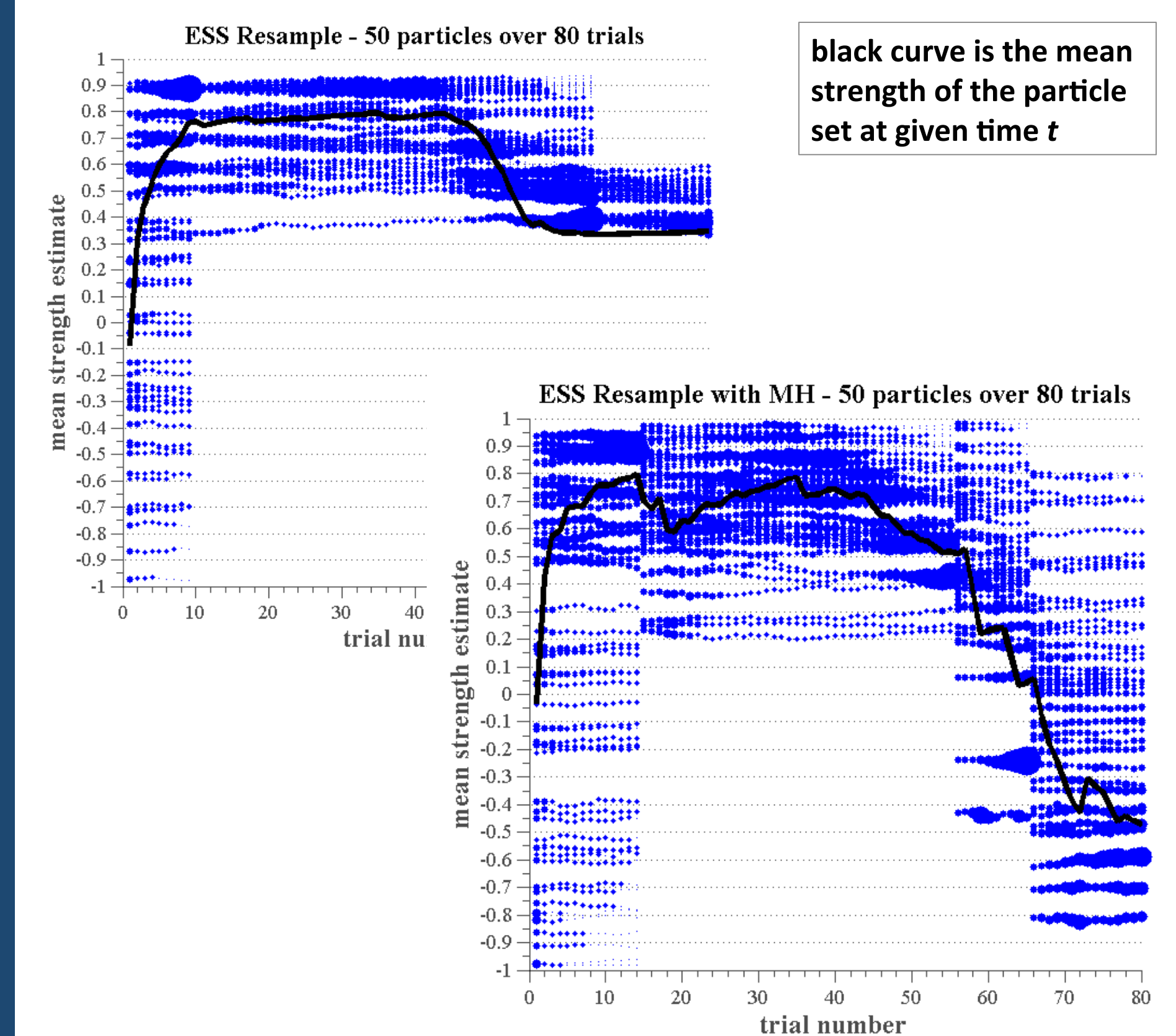


**Model 4:** ESS resample with rejuvenation – same as Model 3 but with ten iterations of Metropolis-Hastings immediately after resampling.

## A Closer Look at Rejuvenation

How are we getting a primacy effect when we don't rejuvenate and a recency effect when we do?

We get a better understanding of the predictions of Models 3 and 4 by focusing on the predictions of 50 particles at each trial  $t$ .



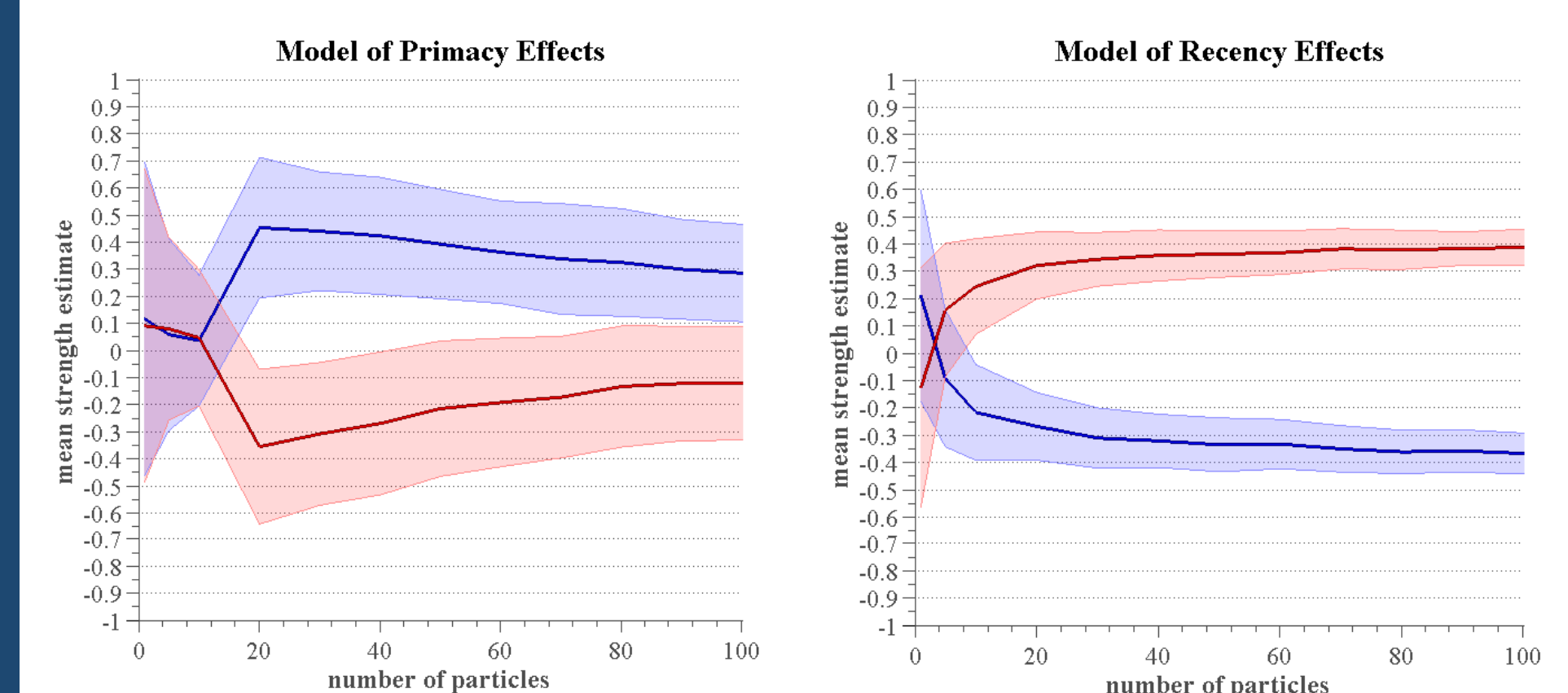
In Model 3, the diversity of the particle set narrows after resampling, resulting in a primacy effect.

In Model 4, the diversity is much broader after the MH rejuvenation step, producing a recency effect.

## Modeling Human Data

Using Model 3 with a stronger prior for generative causes, we obtain results similar to reported primacy effects with a small number of particles.

If we add the MH rejuvenation step after every 10 trials, we additionally obtain results similar to reported recency effects with a small number of particles.



## Conclusions

- Different resampling methods in a particle filter can produce different order effects in a causal learning task and provide a more consistent explanation of observed order effects in behavioral data.
- Two key elements interacting: *filtering*, in which we observe one data point at a time, and *rejuvenation*, in which we consider all previously observed data. This interaction may explain why people produce order effects.

## References

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