

# Modeling order effects in causal learning

josh abbott

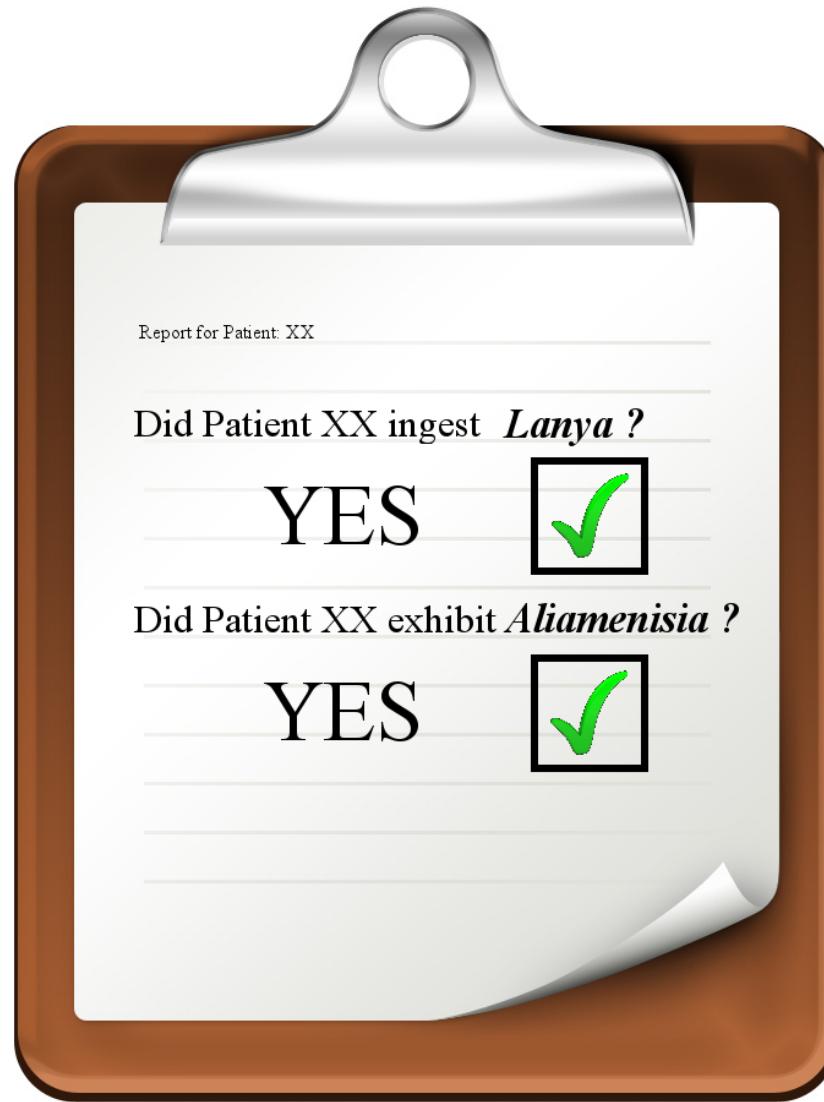
UC Berkeley, Dept. of Psychology

# Outline

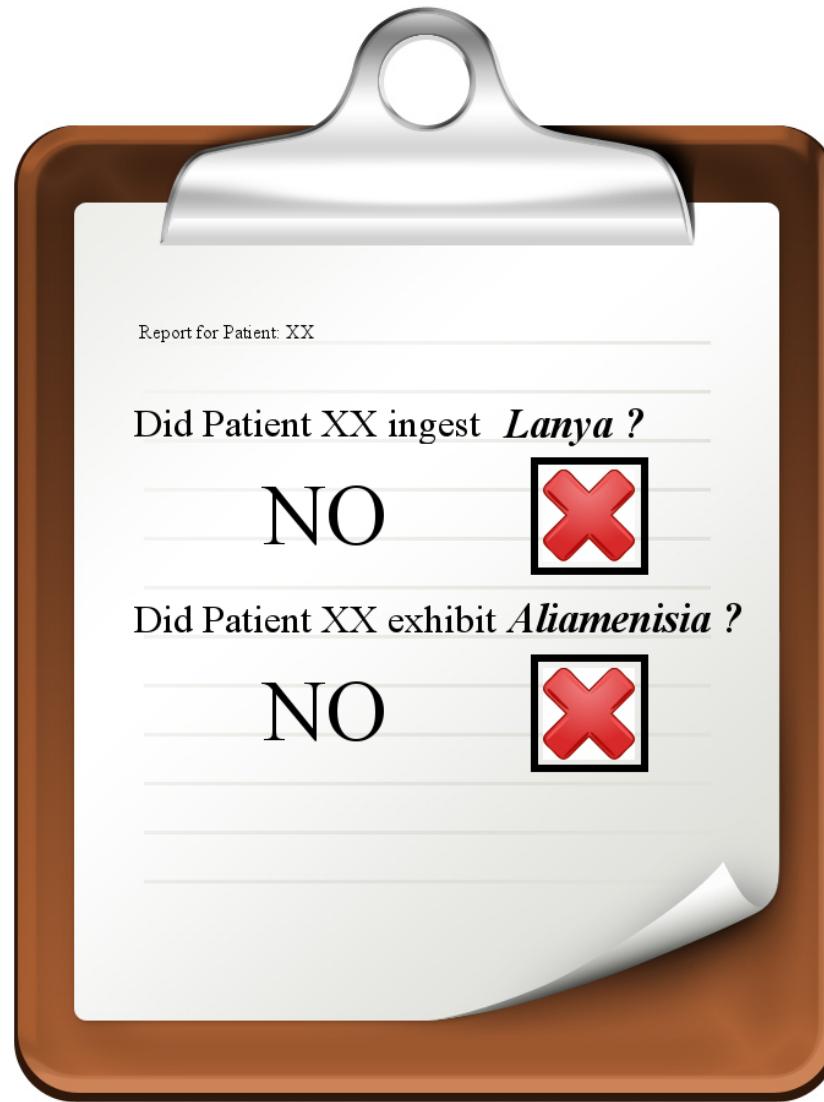
- Order effects (in causal learning)
- Bayesian models (of causal learning)
- Particle filters (for causal learning)

# Order effects in causal learning

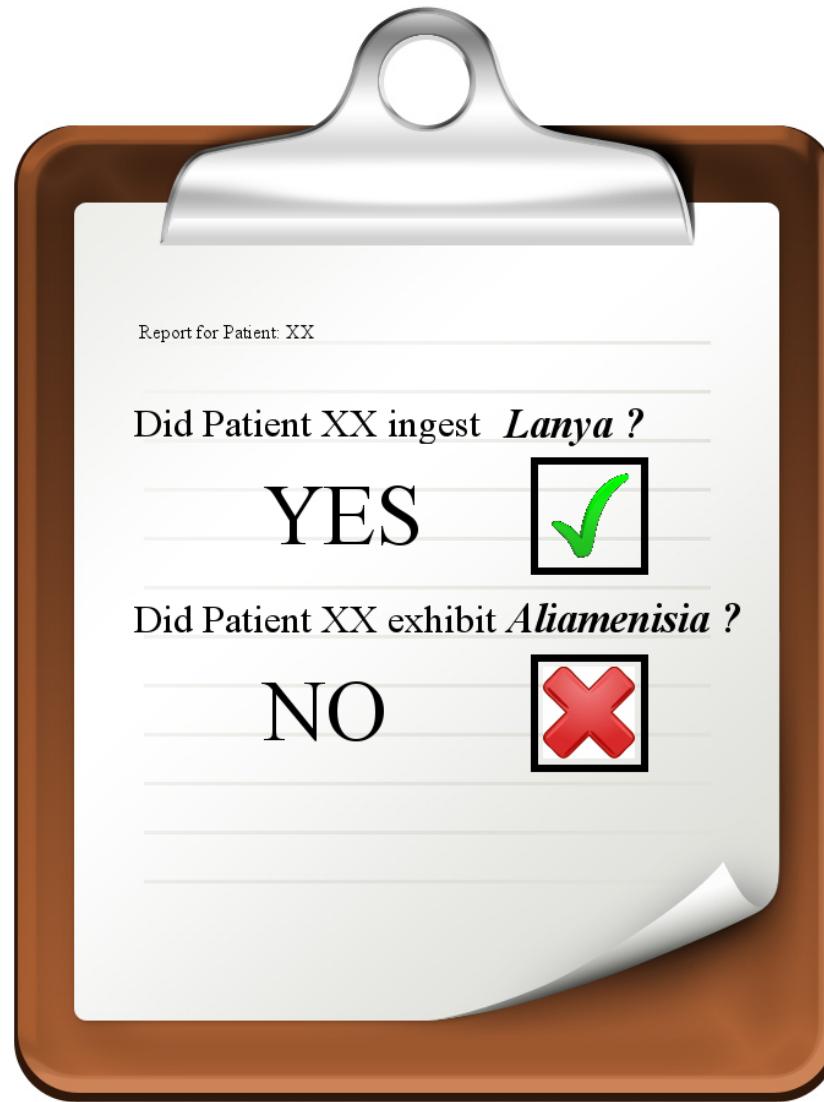
# Order effects in causal learning



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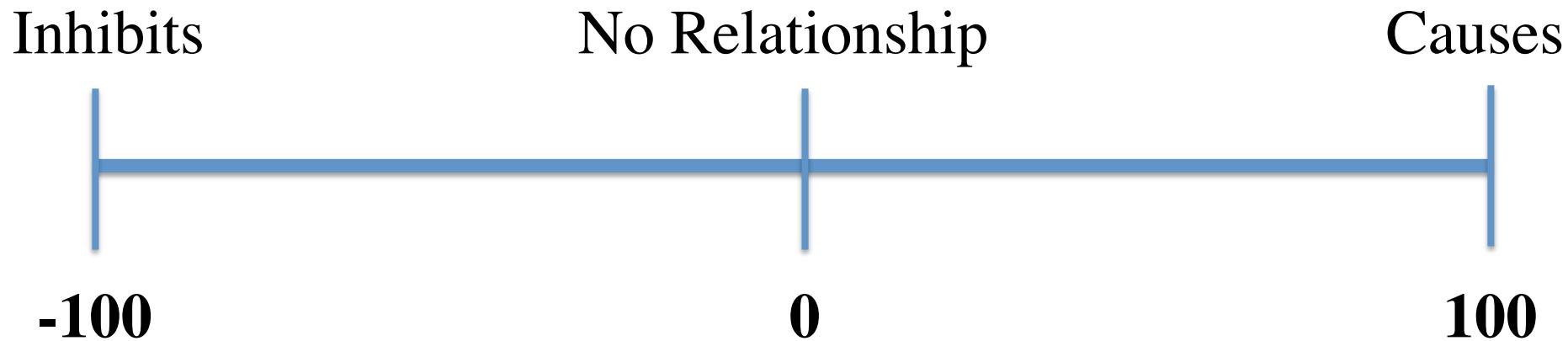


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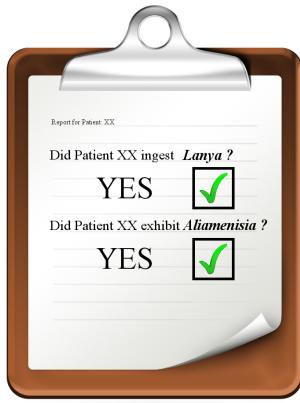
...

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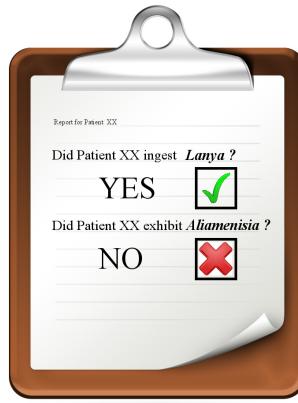
# To what extent does *Lanya* cause *Aliamenisia*?



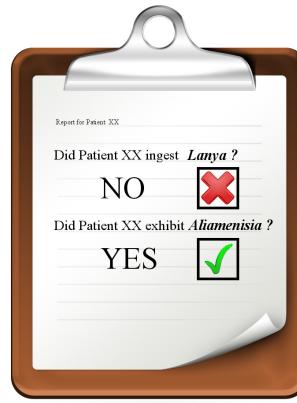
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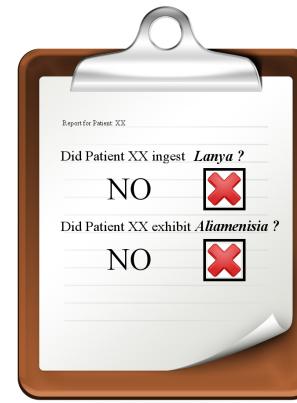
A



B

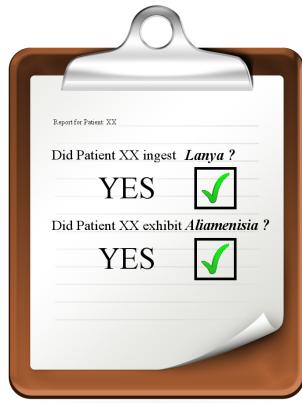


C

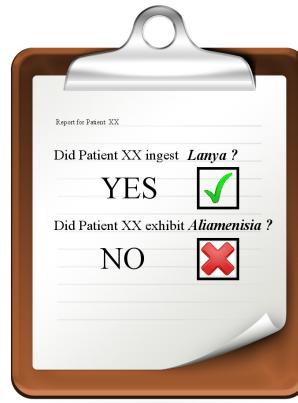


D

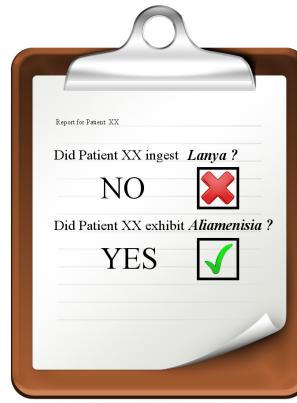
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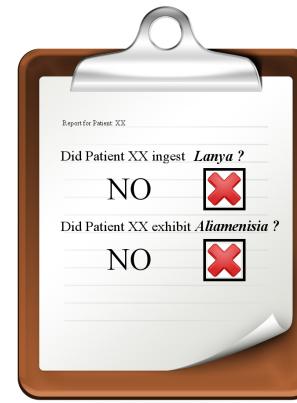
A



B



C

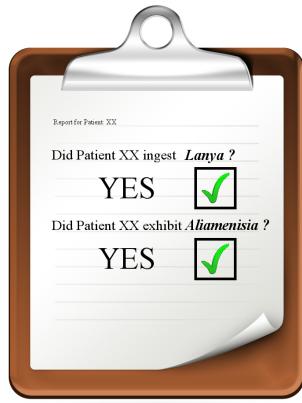


D

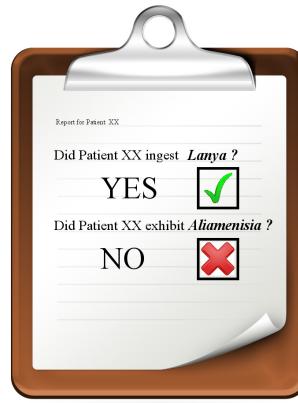
**Contingency Table**

	$E^+$	$E^-$
$C^+$	A	B
$C^-$	C	D

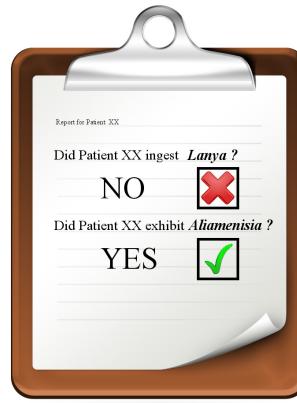
# Order effects in causal learning



A



B



C

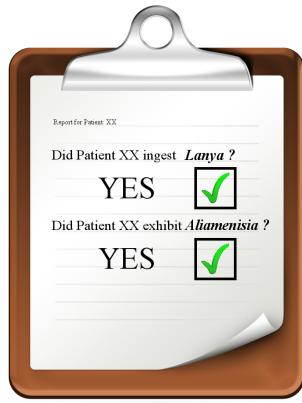


D

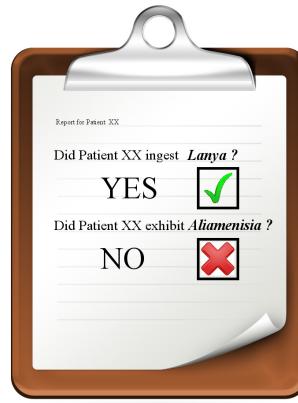
**Contingency Table**

	$E^+$	$E^-$
$C^+$	18	2
$C^-$	2	18

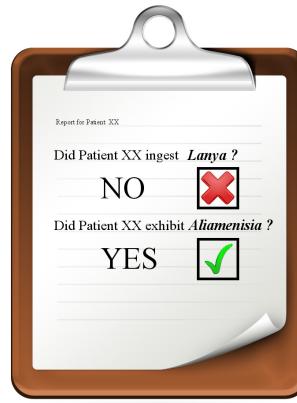
# Order effects in causal learning



A



B



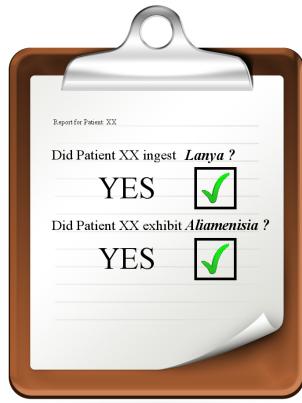
C



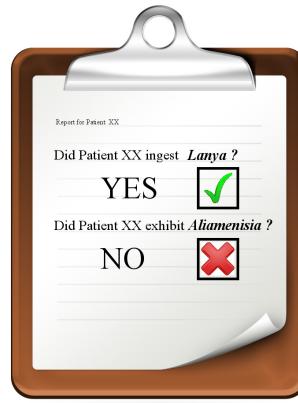
D

Generative		
	$E^+$	$E^-$
$C^+$	18	2
$C^-$	2	18

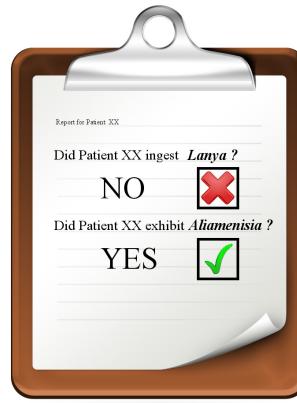
# Order effects in causal learning



A



B



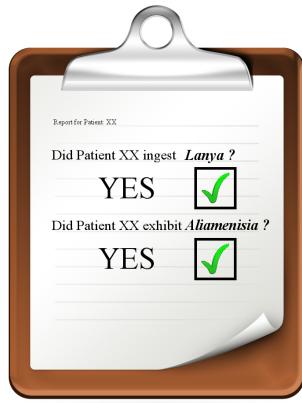
C



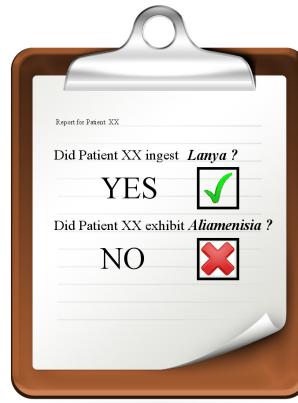
D

Preventative		
	E <sup>+</sup>	E <sup>-</sup>
C <sup>+</sup>	2	18
C <sup>-</sup>	18	2

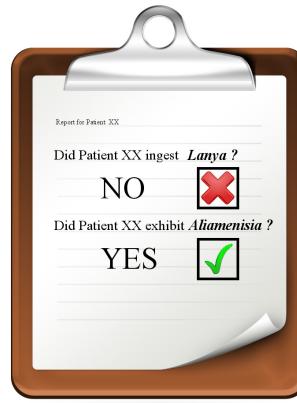
# Order effects in causal learning



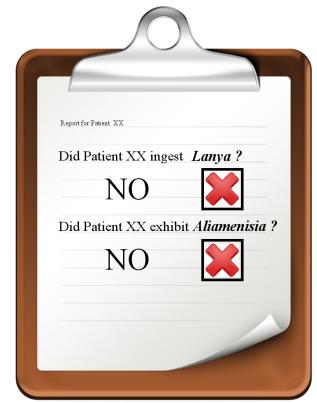
A



B



C

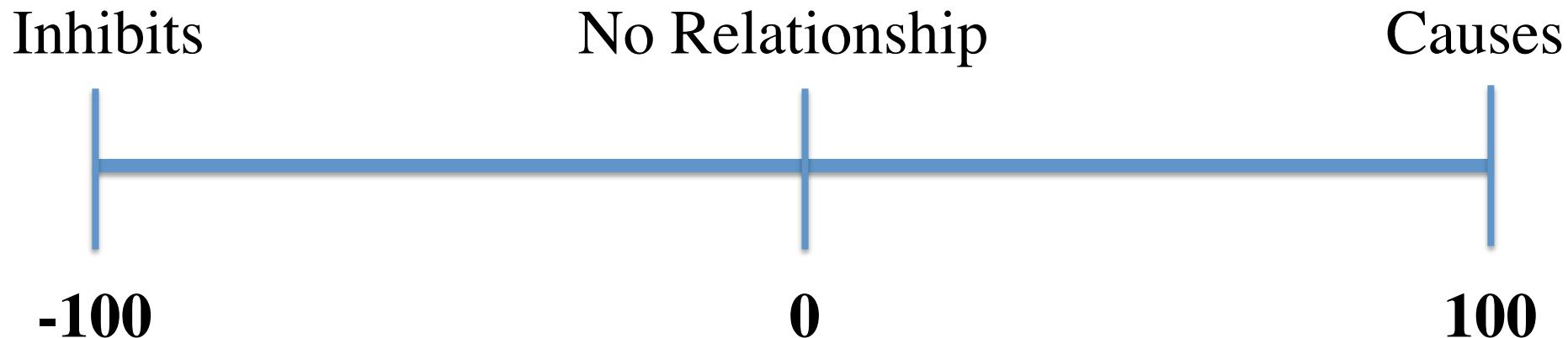


D

Generative Block			Preventative Block		
	E <sup>+</sup>	E <sup>-</sup>		E <sup>+</sup>	E <sup>-</sup>
C <sup>+</sup>	18	2	C <sup>+</sup>	2	18
C <sup>-</sup>	2	18	C <sup>-</sup>	18	2

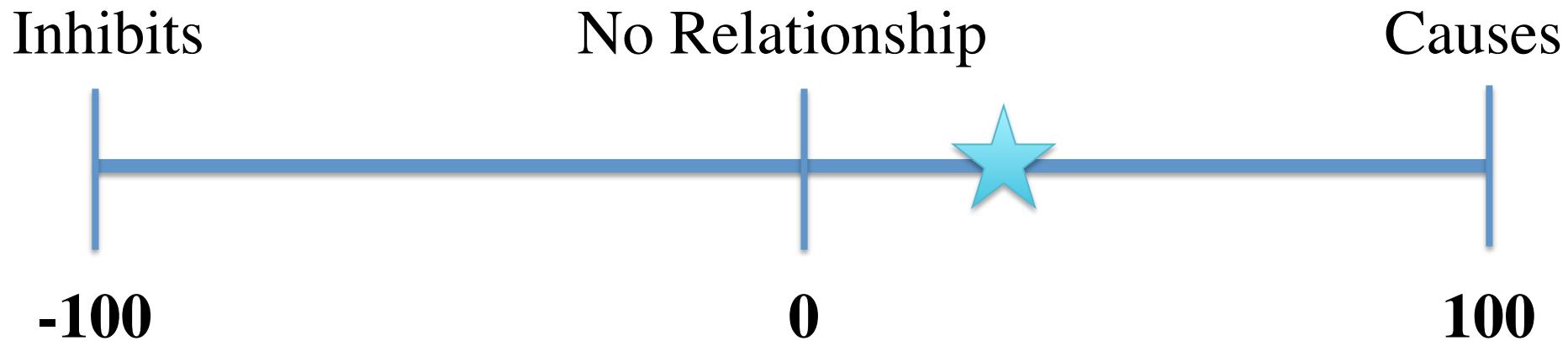
# Order effects in causal learning

# To what extent does *Lanya* cause *Aliamenisia*?



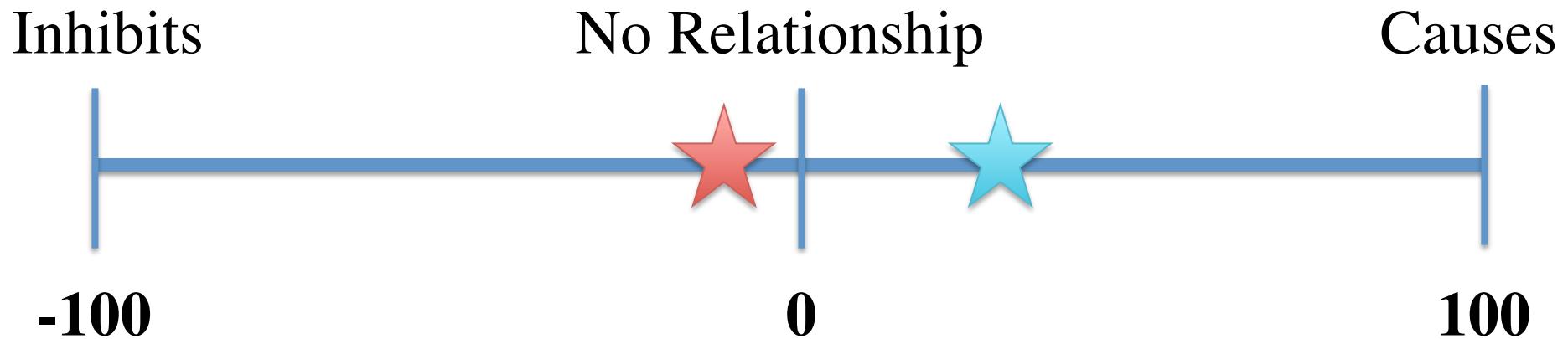
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# Order effects in causal learning

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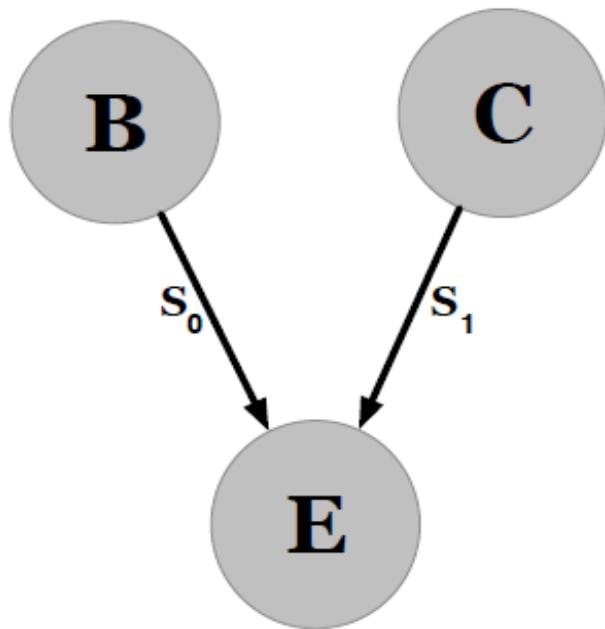
**Primacy effects:** initial information has greatest impact on later judgments.

# Order effects in causal learning

**Primacy effects:** initial information has greatest impact on later judgments.

**Recency effects:** most recent information has greatest impact on later judgments.

# Bayesian model of causal learning



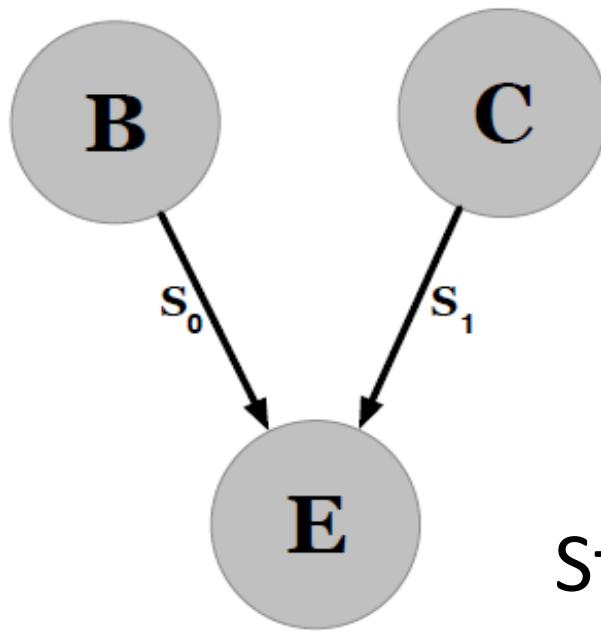
3 binary variables:

(E) Effect

(C) Potential cause of interest

(B) Background causes

# Bayesian model of causal learning



3 binary variables:

(E) Effect

(C) Potential cause of interest

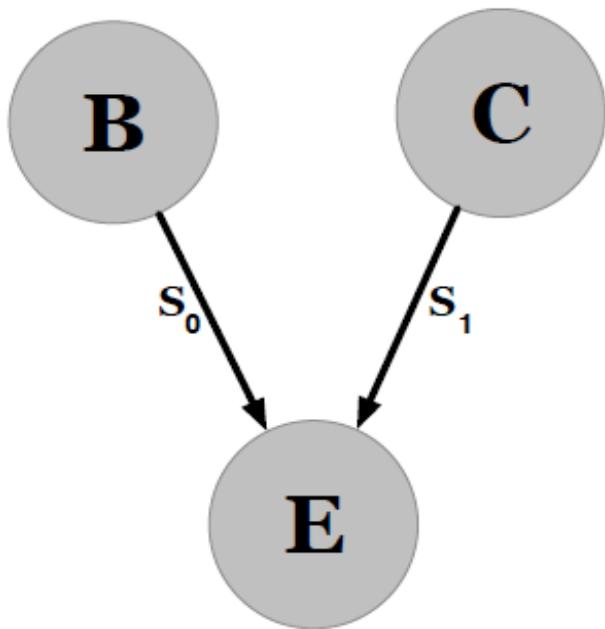
(B) Background causes

Strength estimates on the edges:

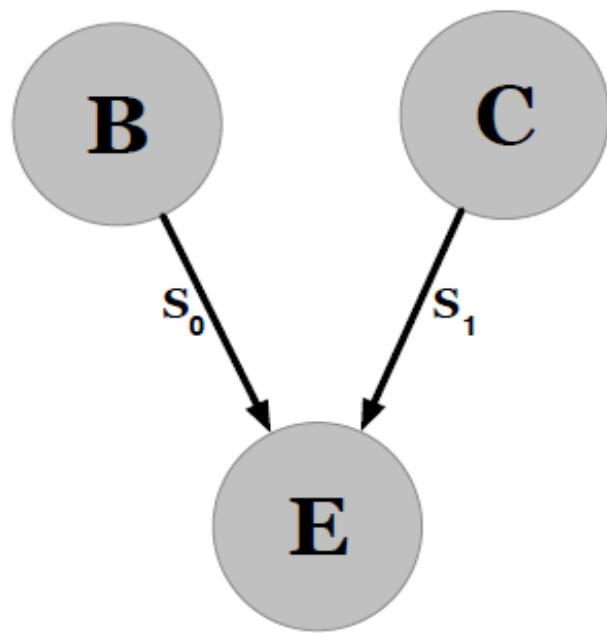
( $s_0, s_1$ ) indicating how strongly  
**B** and **C** influence **E**

# Bayesian model of causal learning

$$P(h | d) \propto P(d | h)P(h)$$



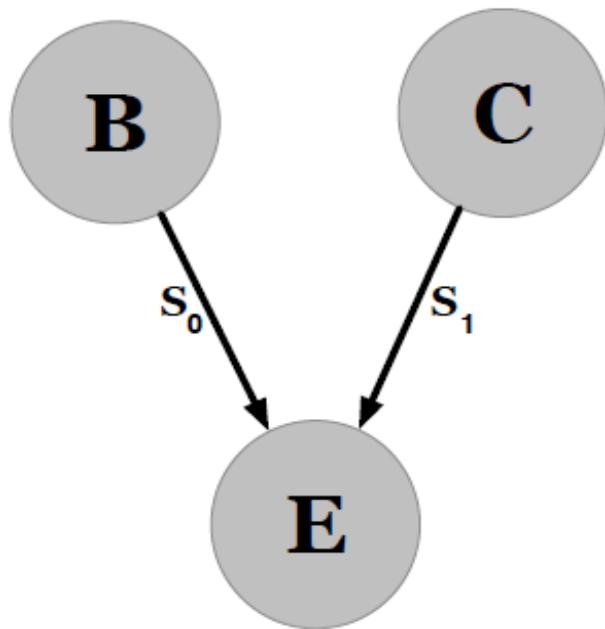
# Bayesian model of causal learning



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Prior:  $P(s_0), P(s_1)$

# Bayesian model of causal learning

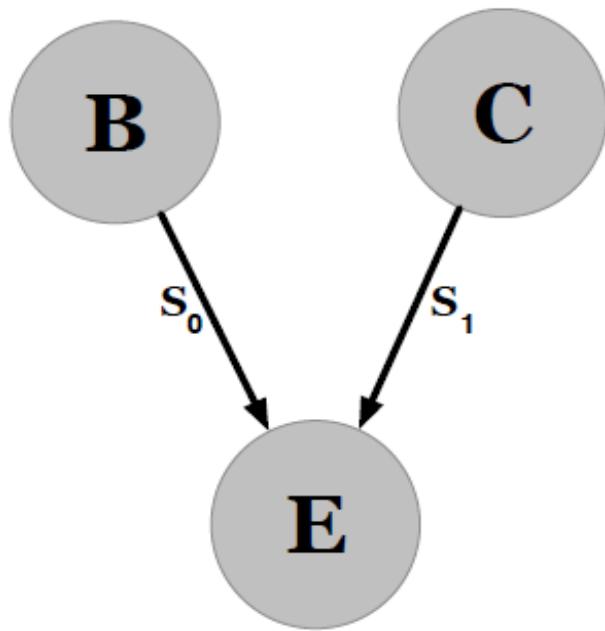


$$P(h | d) \propto P(d | h)P(h)$$

Prior:  $P(s_0), P(s_1)$

Likelihood:  $P(E | B, C; s_0, s_1)$

# Bayesian model of causal learning



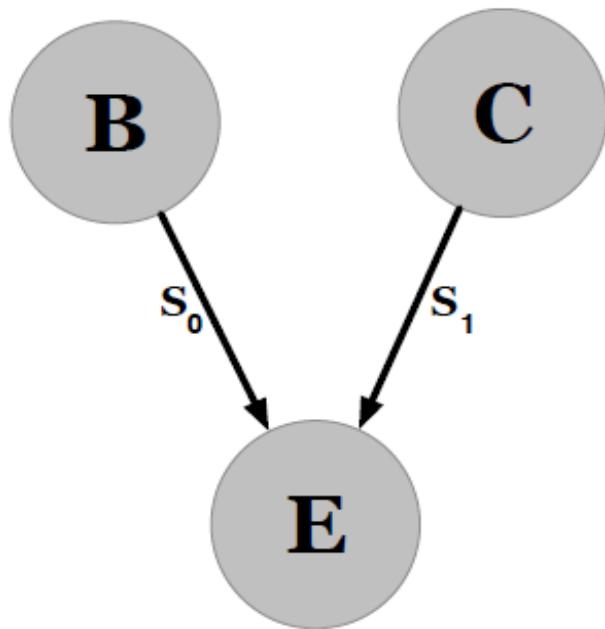
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“Yesterday’s posterior is today’s prior”

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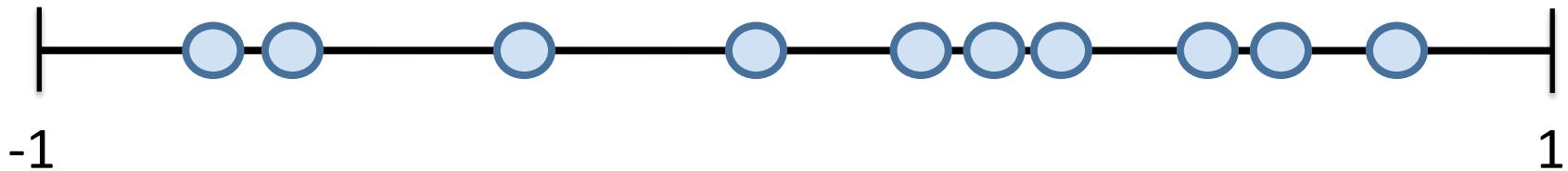
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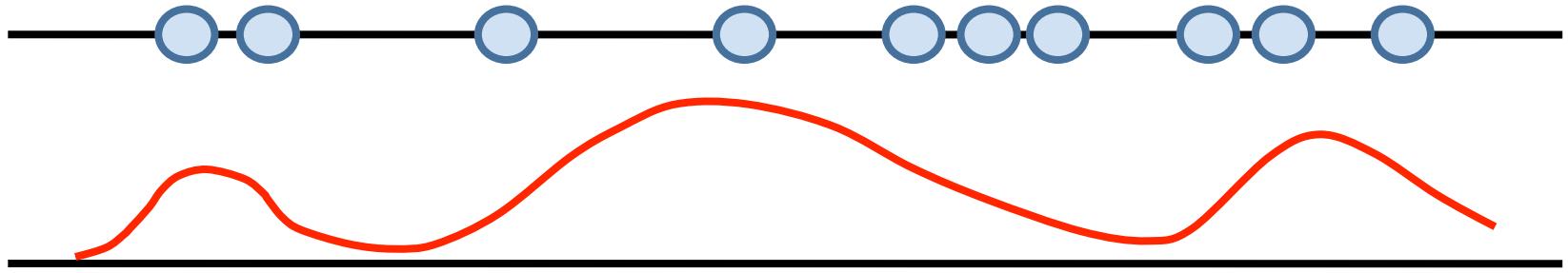
# Particle filters



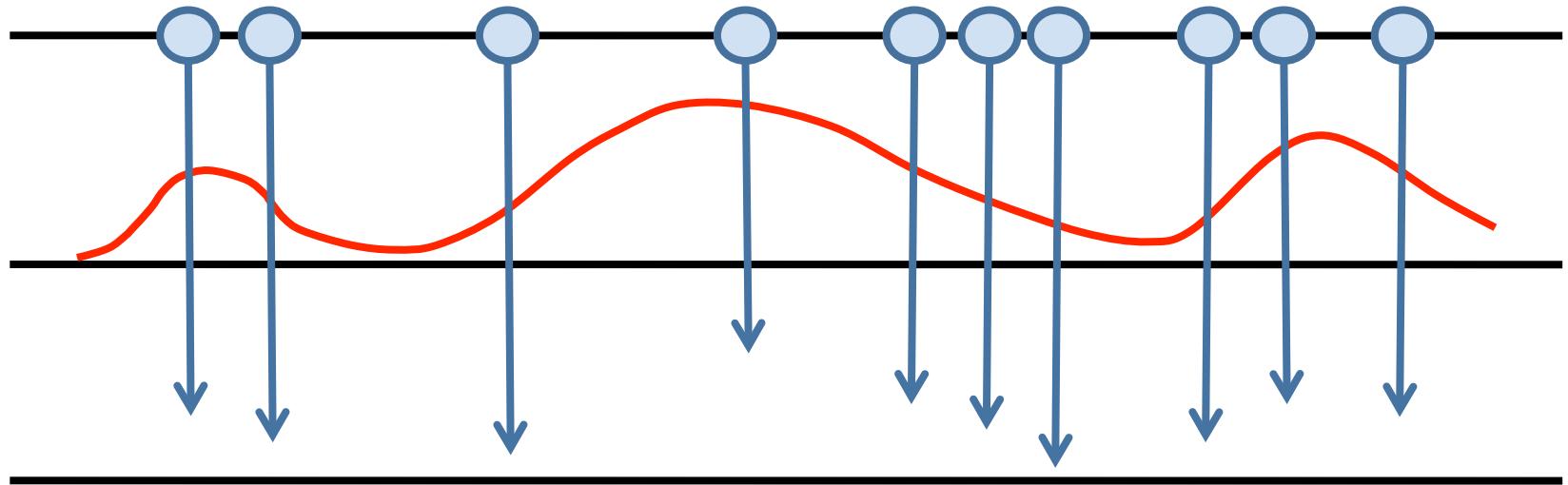
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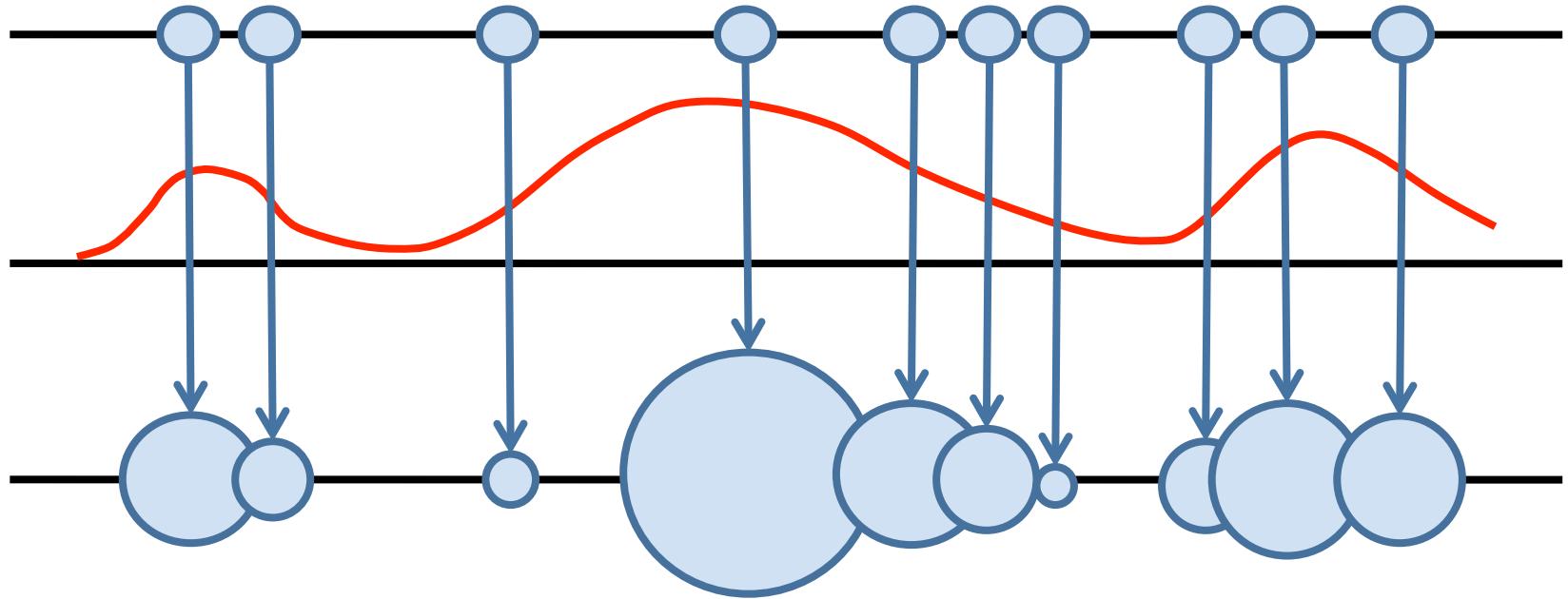
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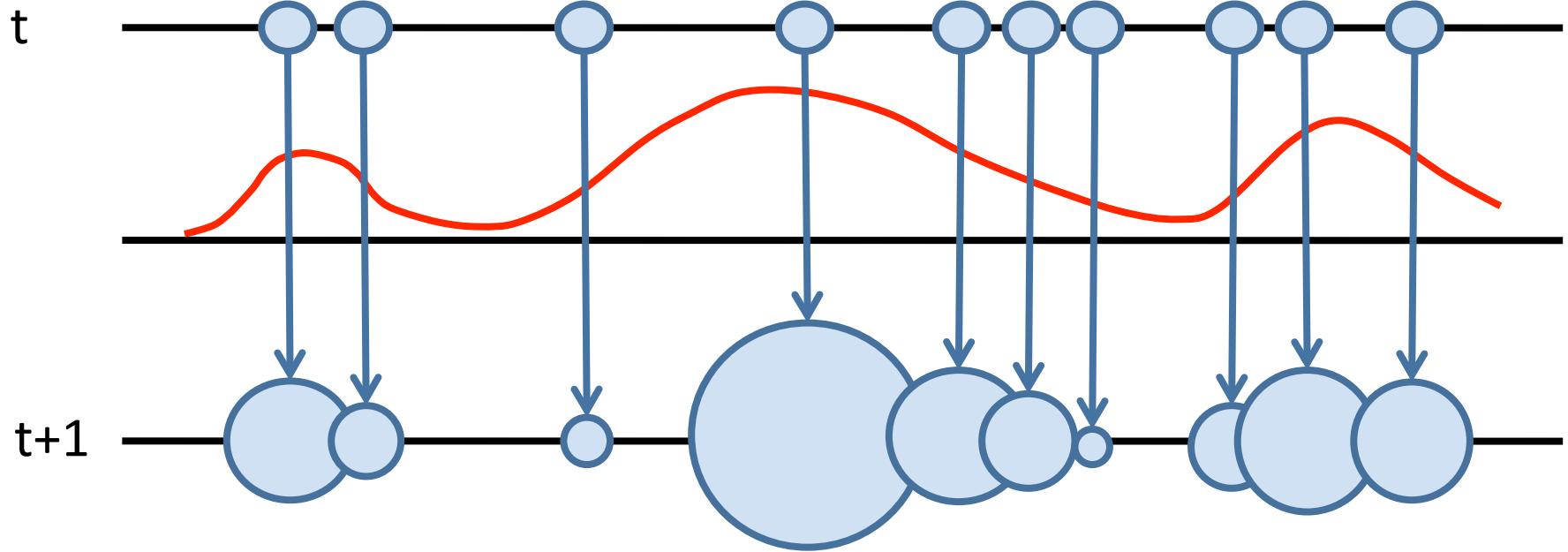


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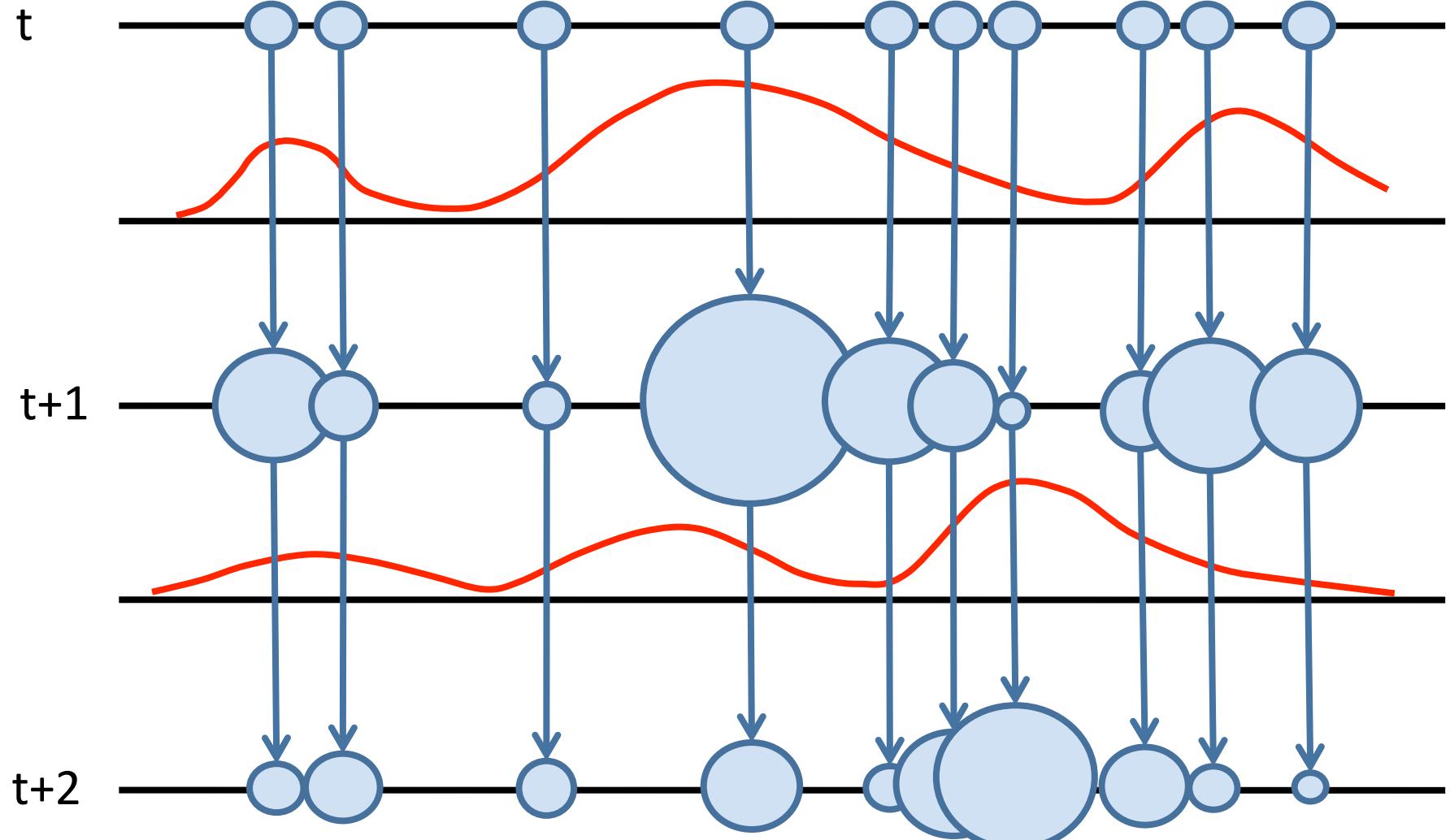
# Particle filters

time:



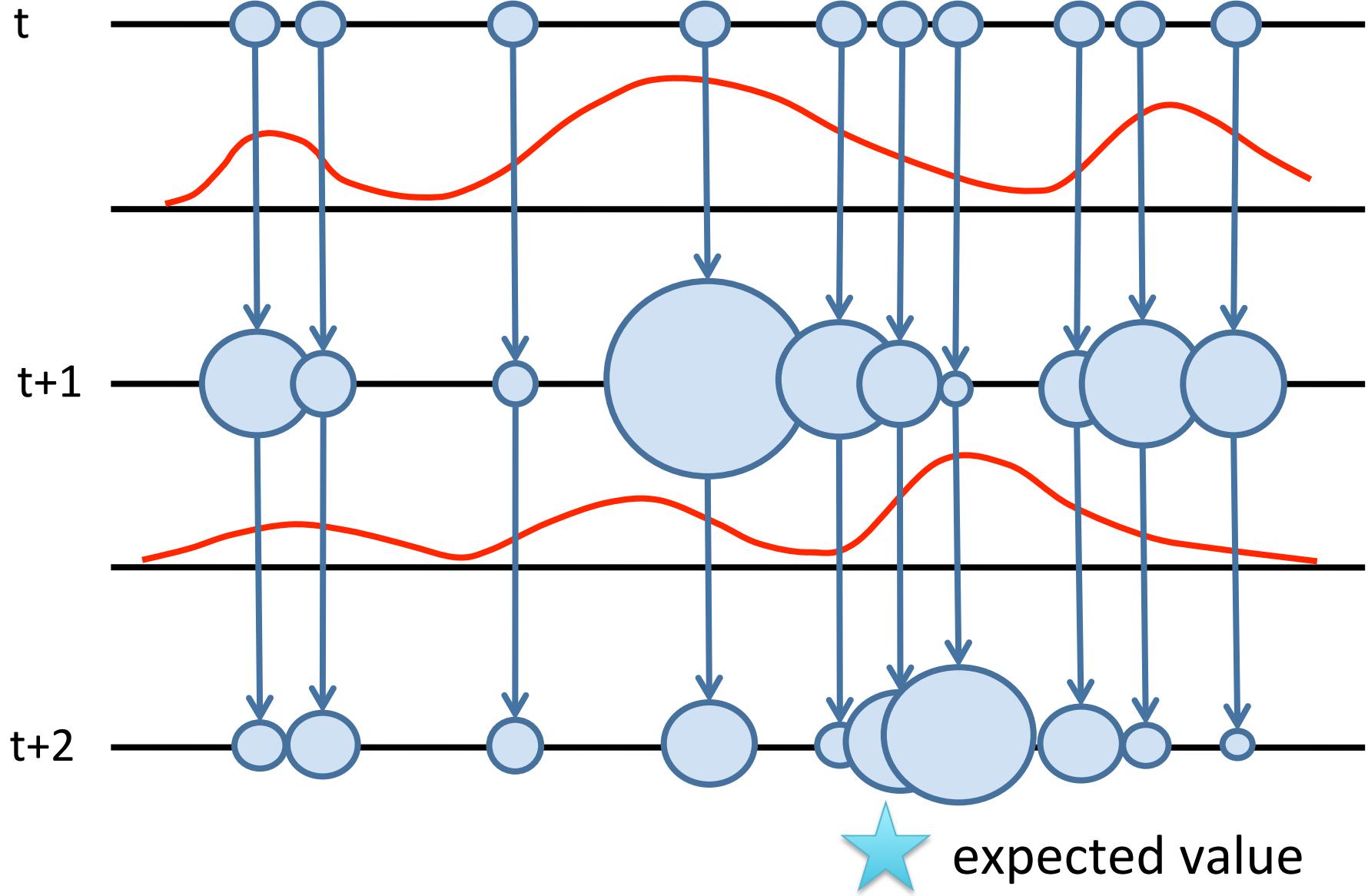
# Particle filters

time:



# Particle filters

time:



# Our particle filter

A particle holds a pair of strength estimates ( $s_0, s_1$ )

Each trial, weight the importance of a particle  
using the noisy-OR and noisy-AND-NOT functions

# Our particle filter

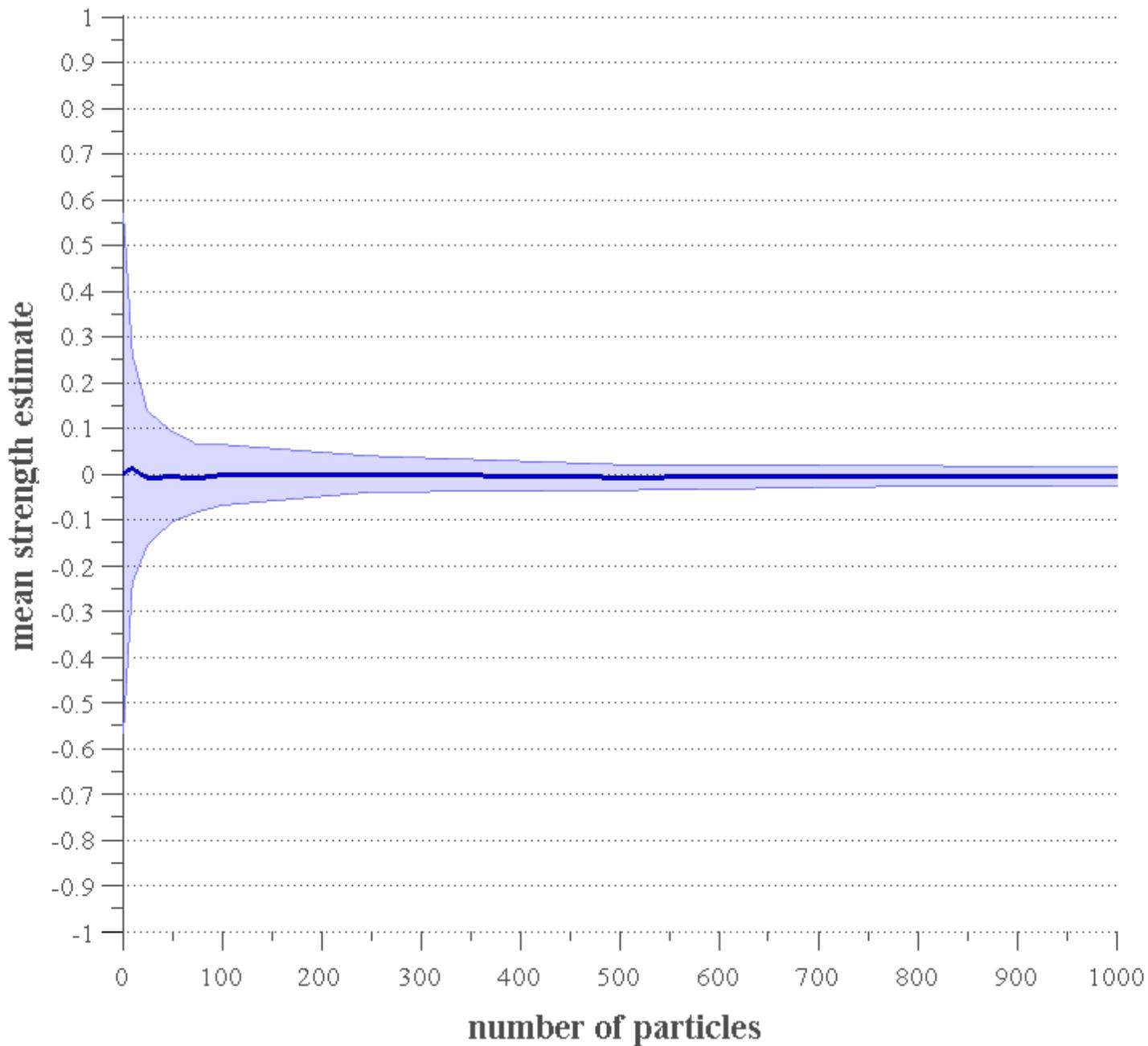
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## Particle filter for causal learning

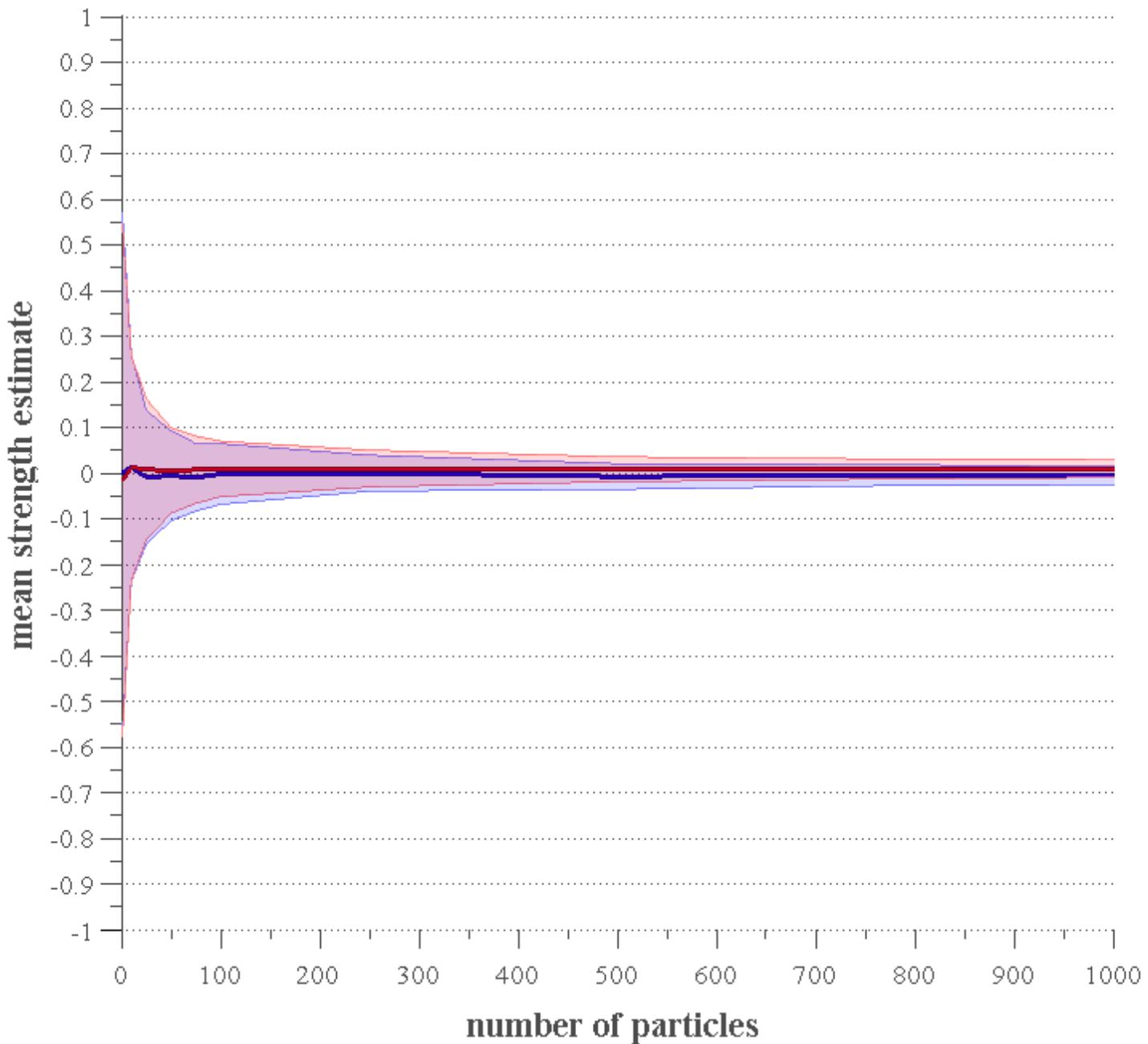


generative  
block first



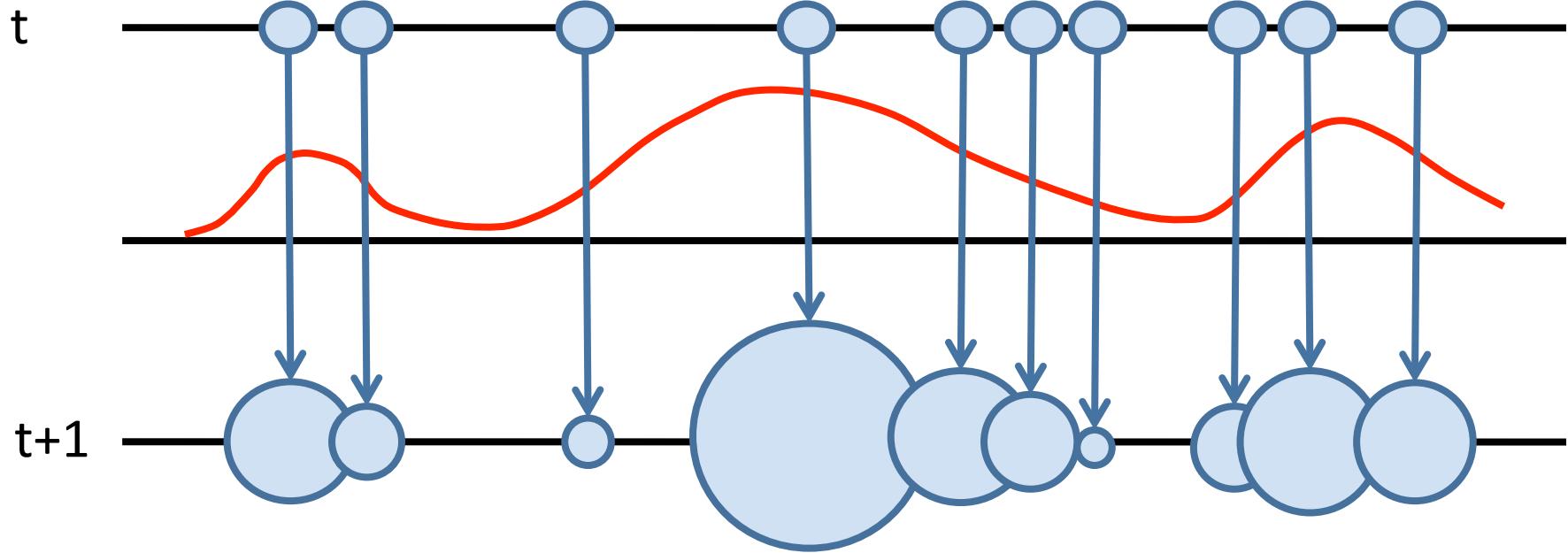
## Particle filter for causal learning

- generative block first
- preventative block first



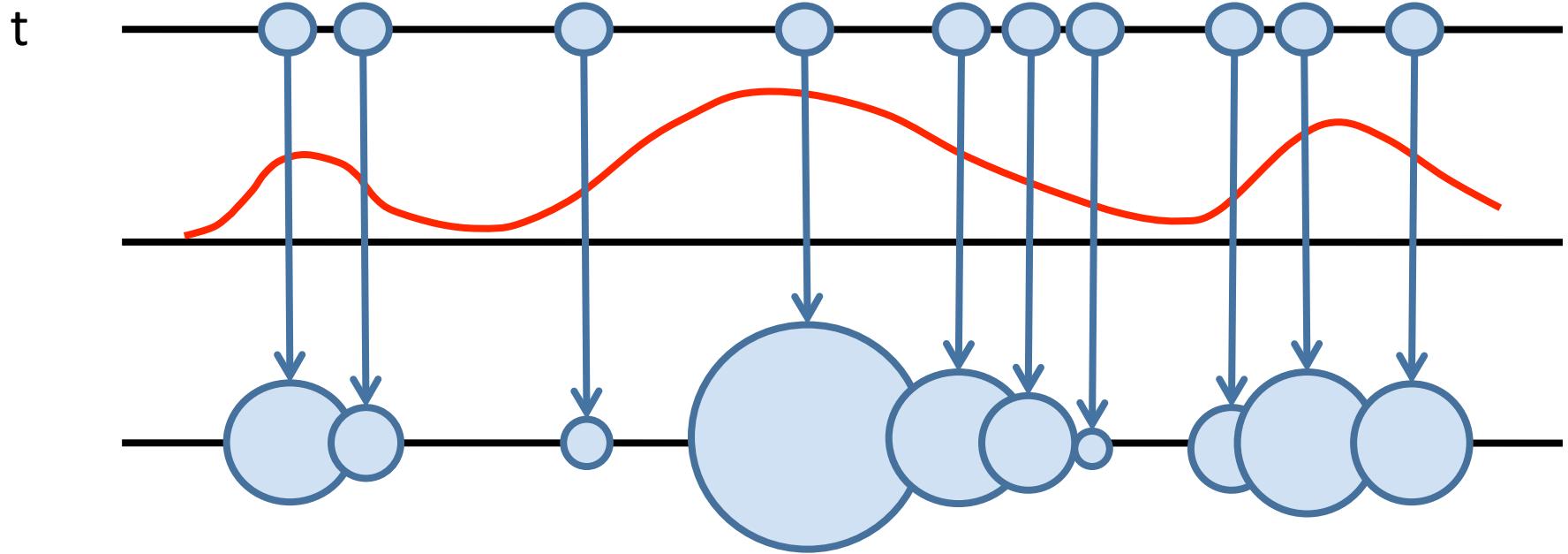
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time:



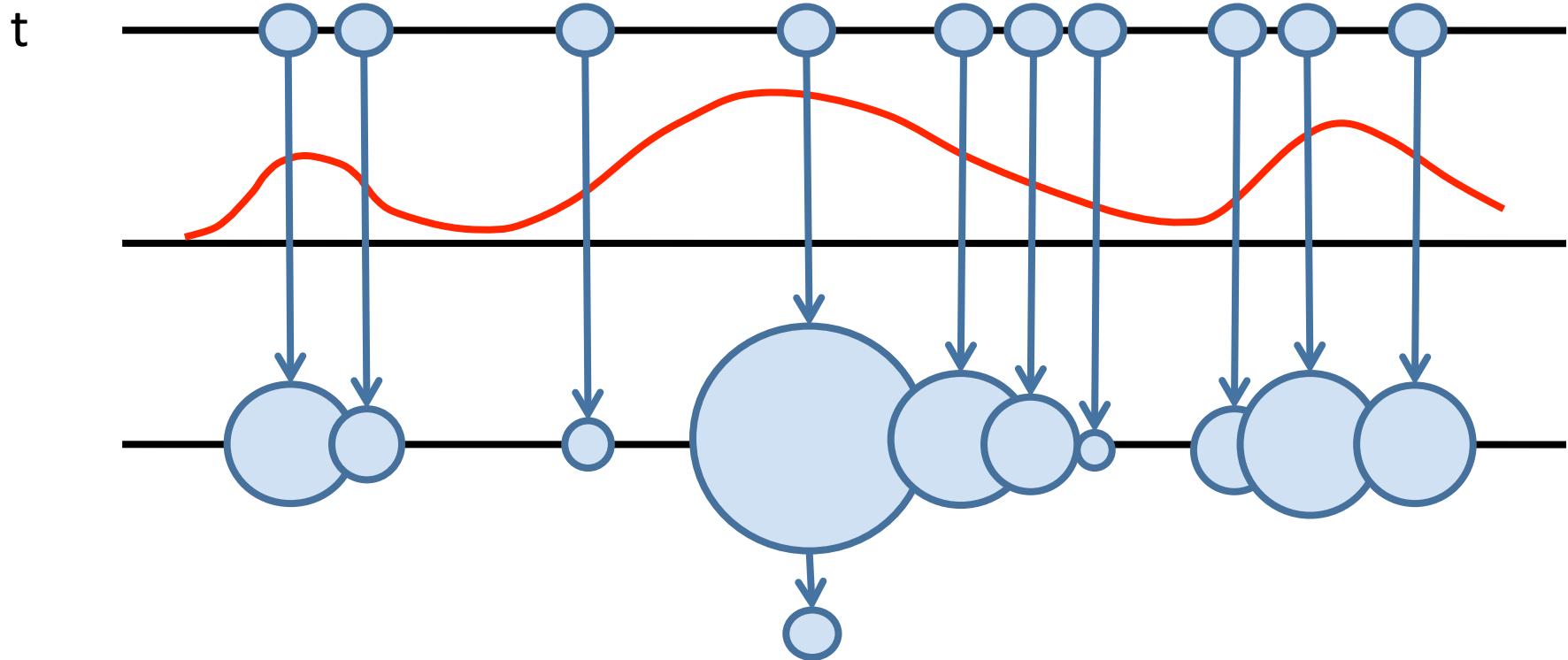
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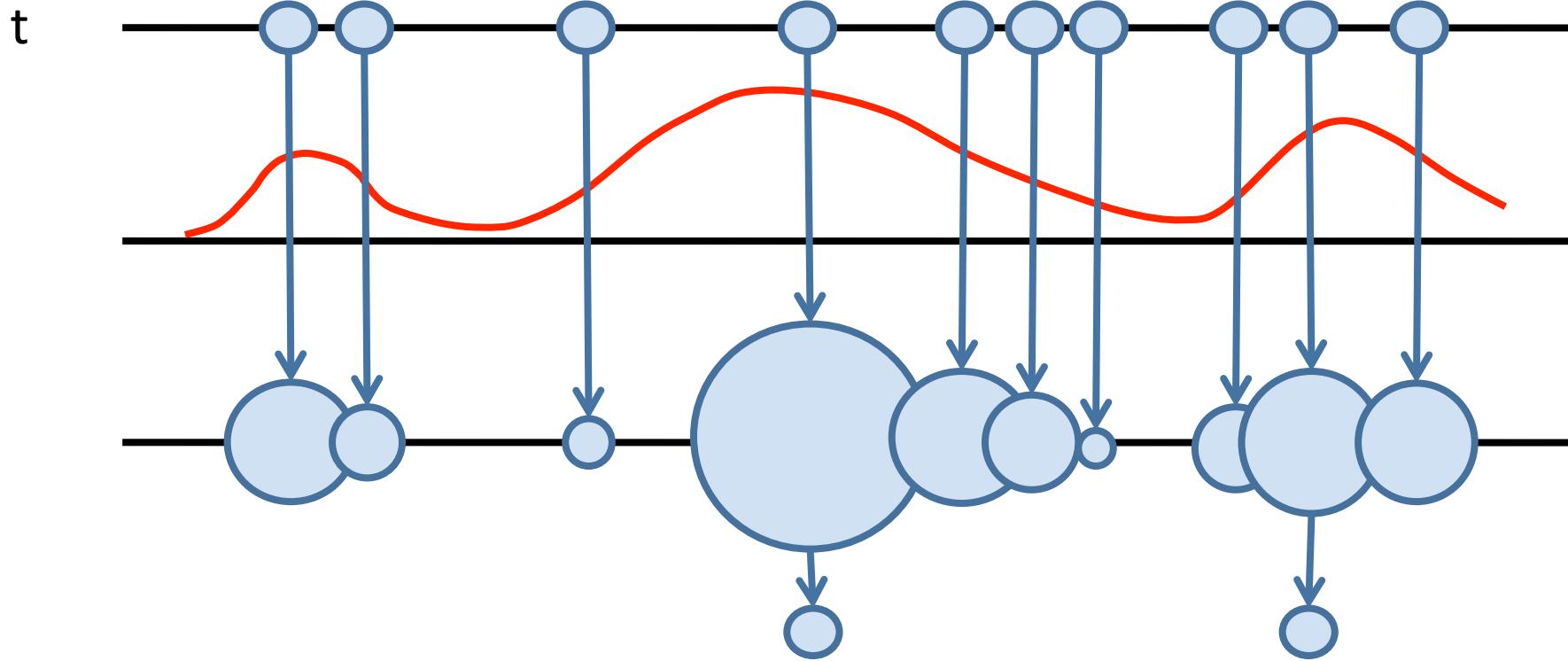
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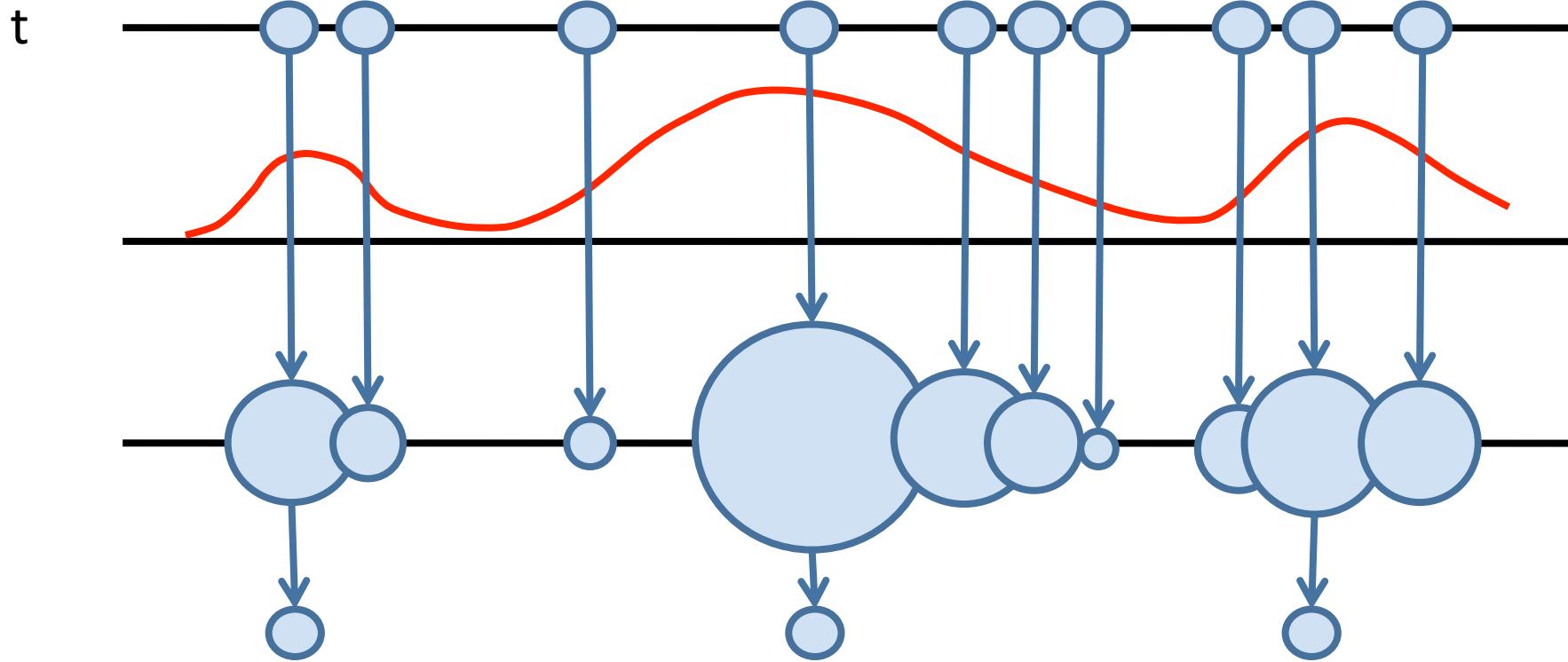
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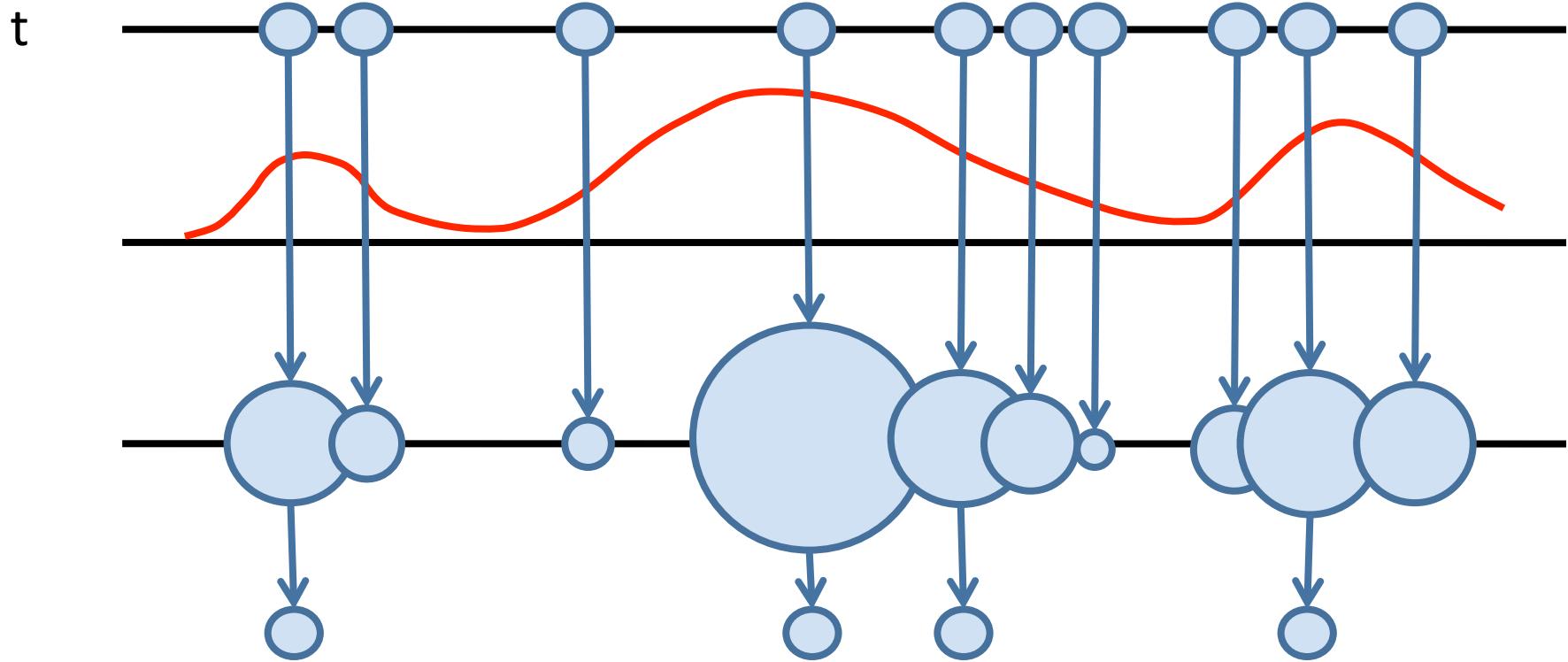
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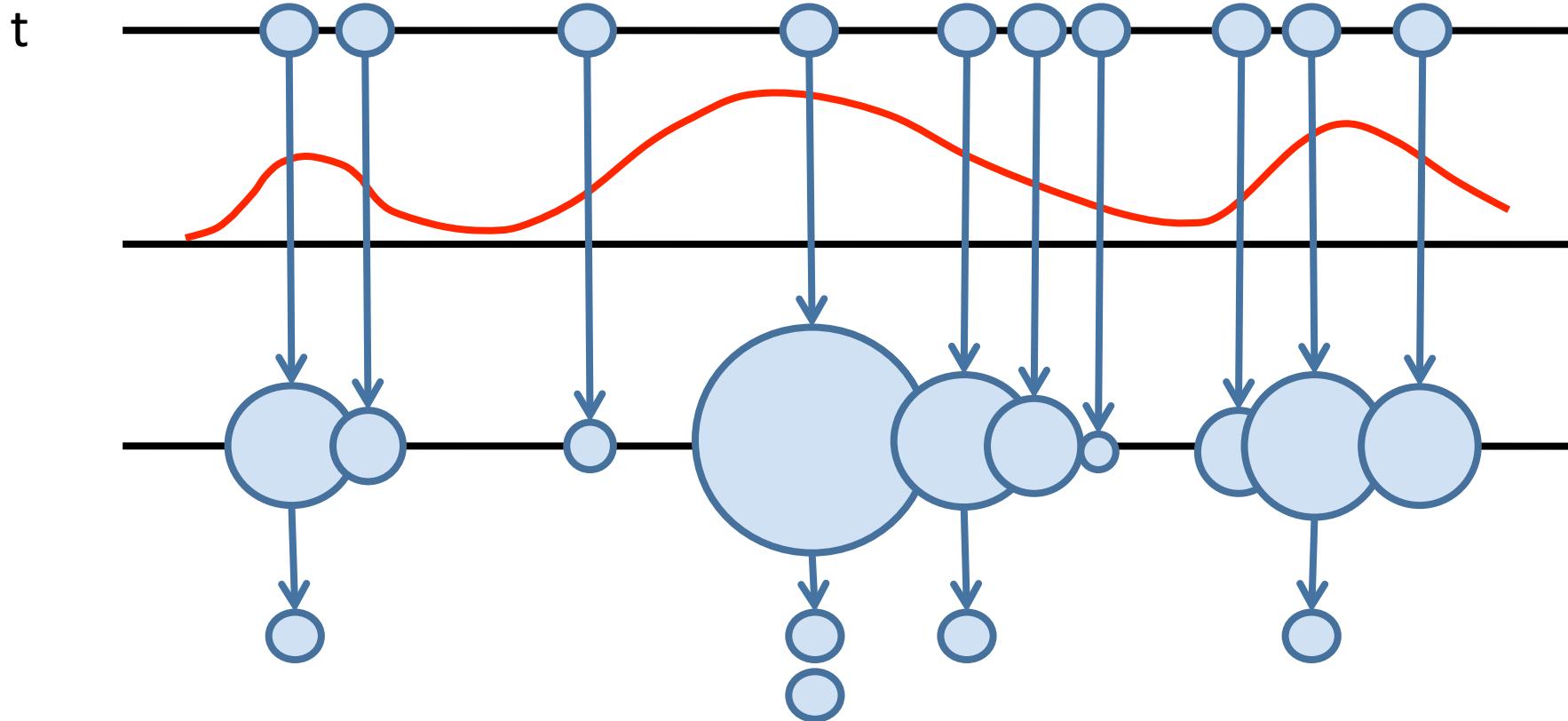
# Particle filters

time:



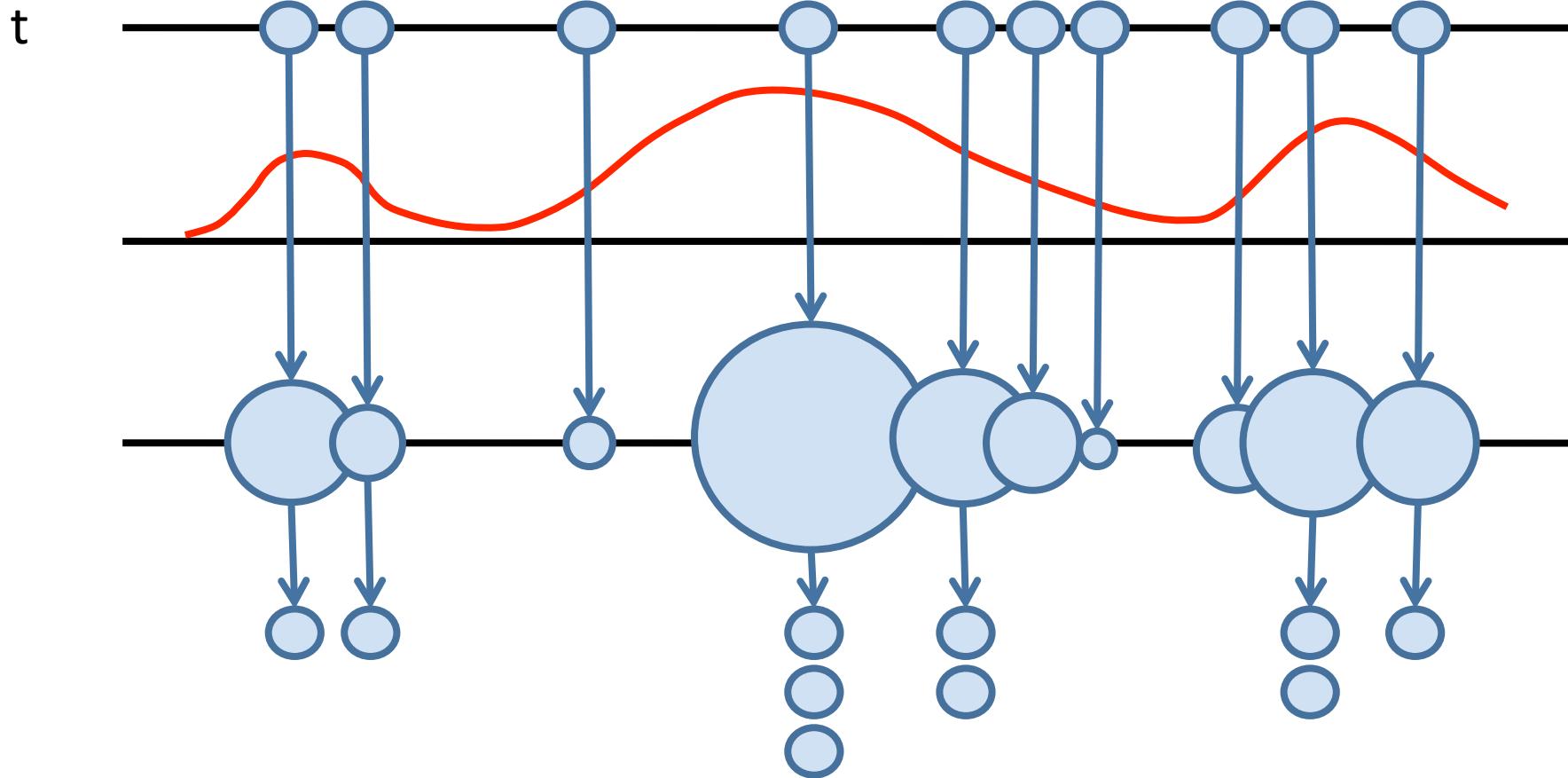
# Particle filters

time:



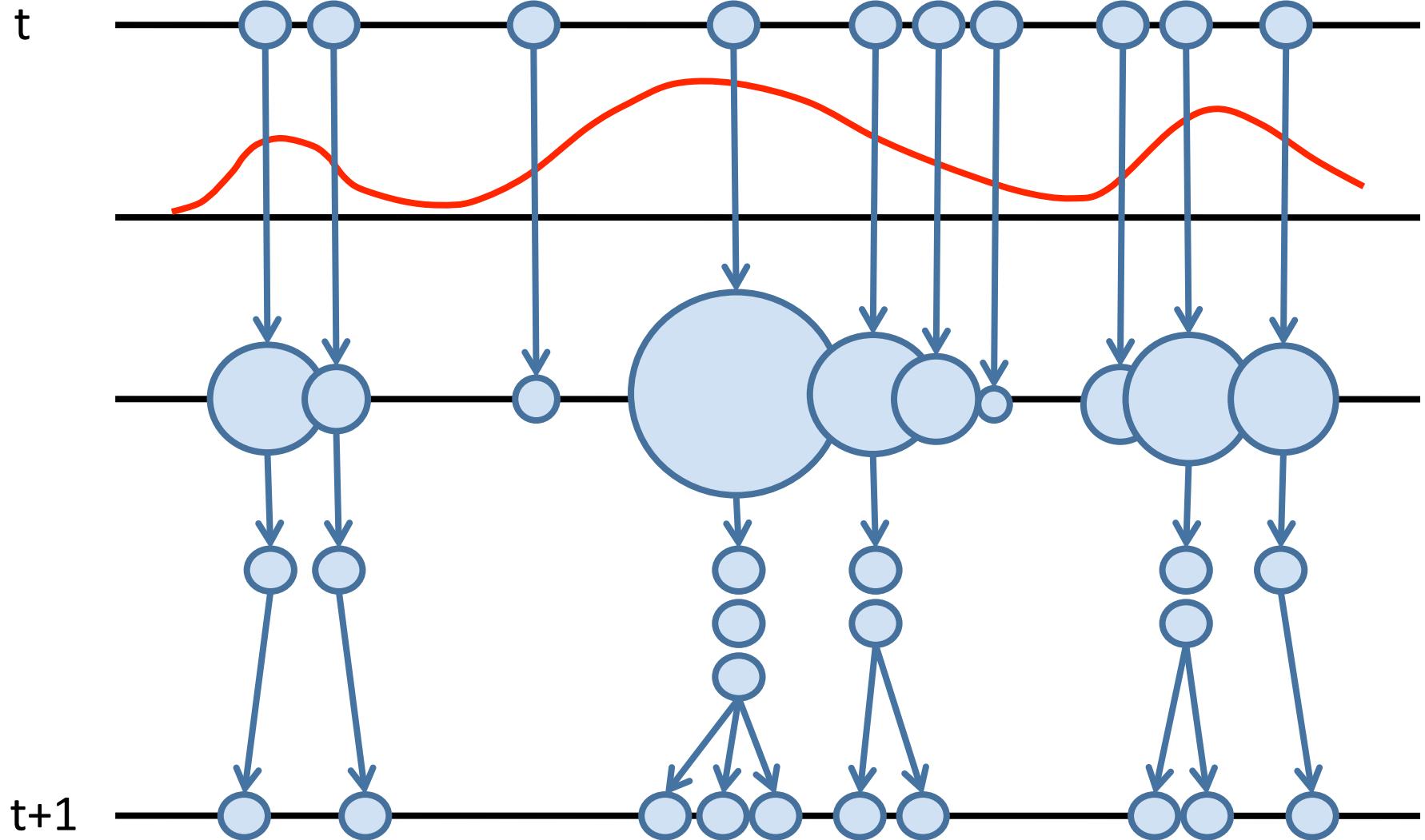
# Particle filters

time:



# Particle filters

time:



# Our particle filter

A particle holds a pair of strength estimates ( $s_0, s_1$ )

Each trial, weight the importance of a particle  
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Resample particles using a multinomial distribution  
over the importance weights

Generate new set of particles using a Beta  
distribution over previous particle values

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# Our particle filter

How do different resampling schemes affect the predictions of order effects?

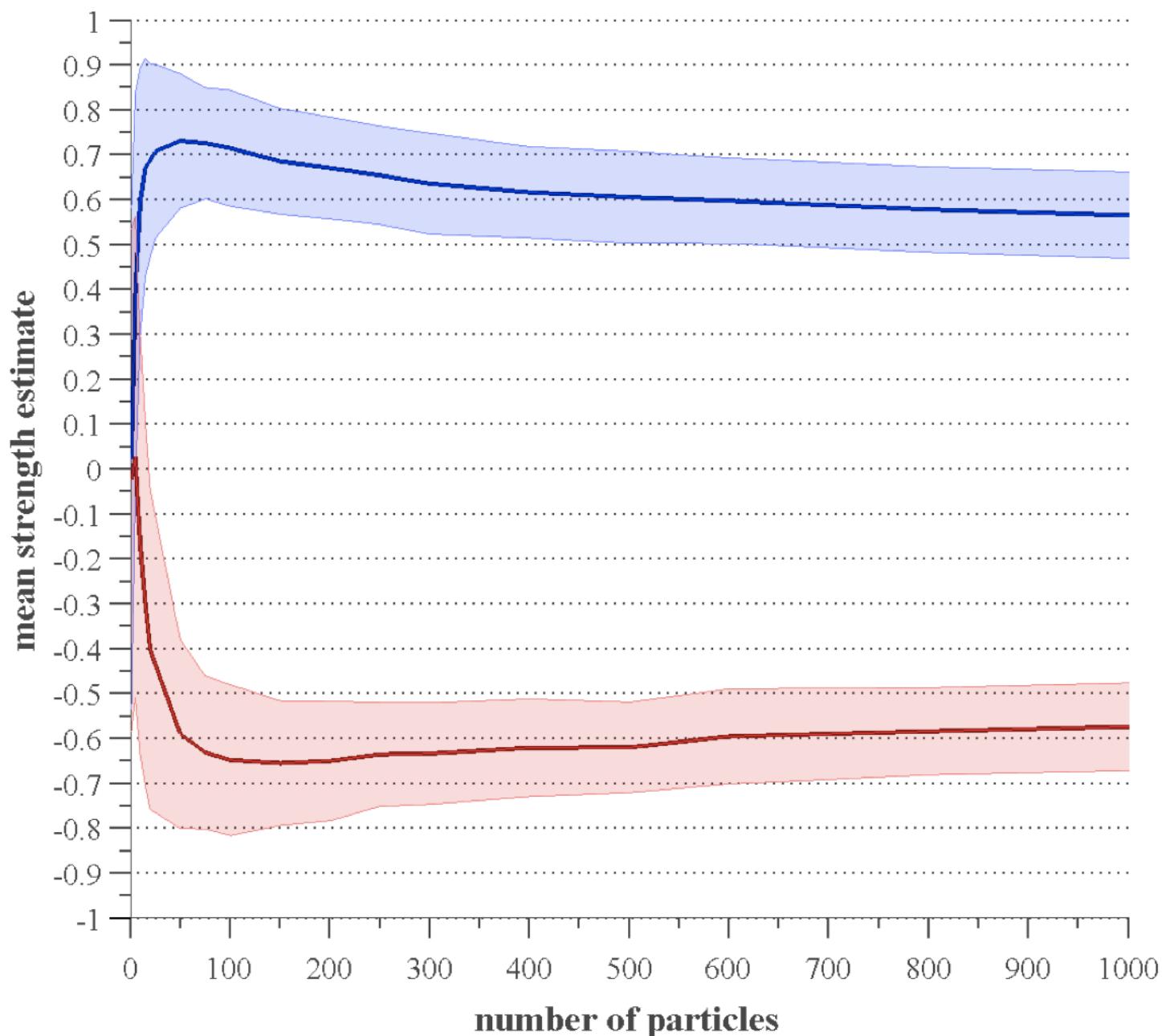
# Model 1: Always resample

Resample after every trial using a multinomial distribution defined on the particle importance weights

## Always Resample

generative  
block first

preventative  
block first

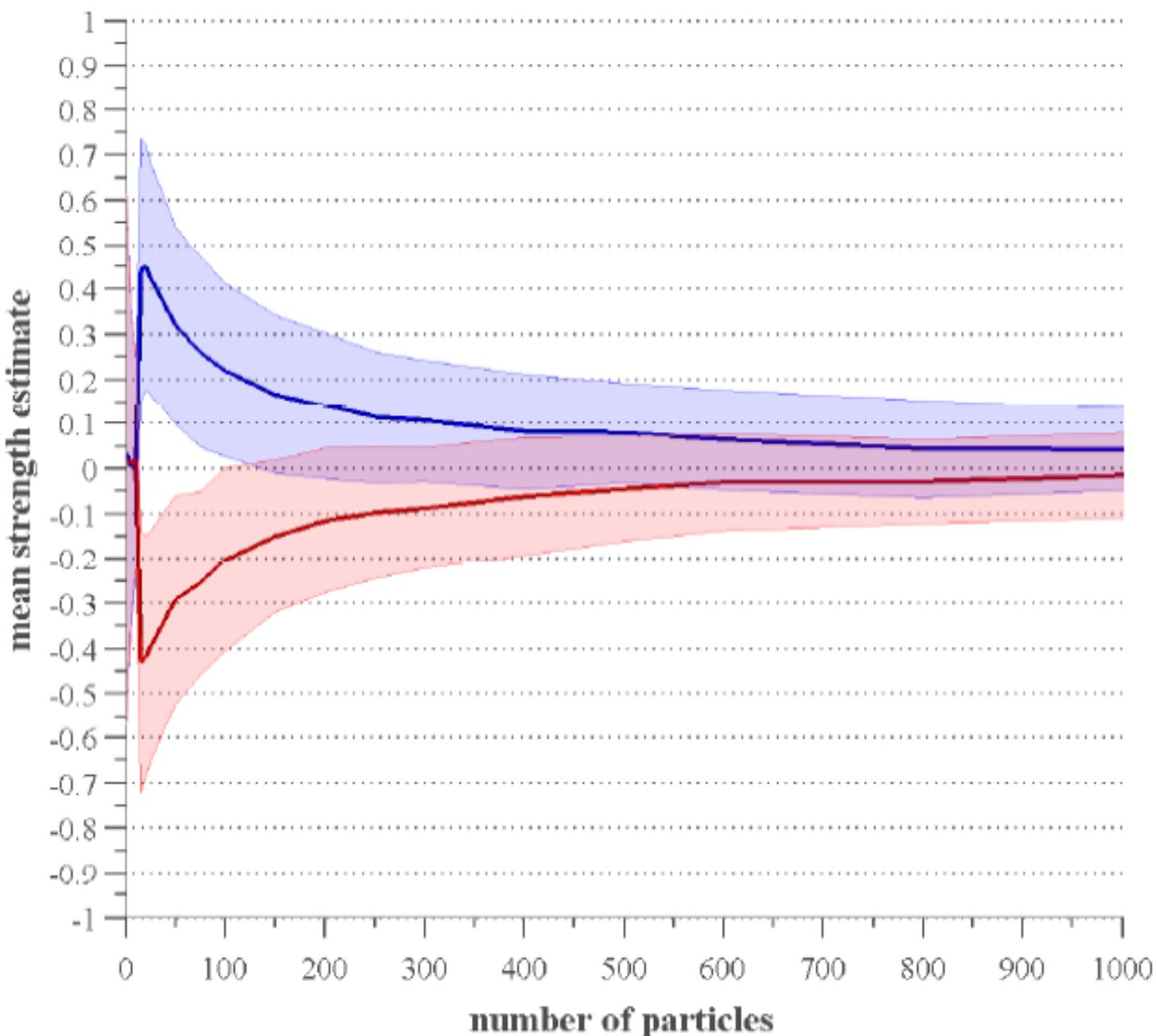


# Model 2: ESS Resample

Resample only if the variance of the importance weights is too large as defined by the Effective Sample Size (ESS)

## ESS Resample

- generative  
block first
- preventative  
block first

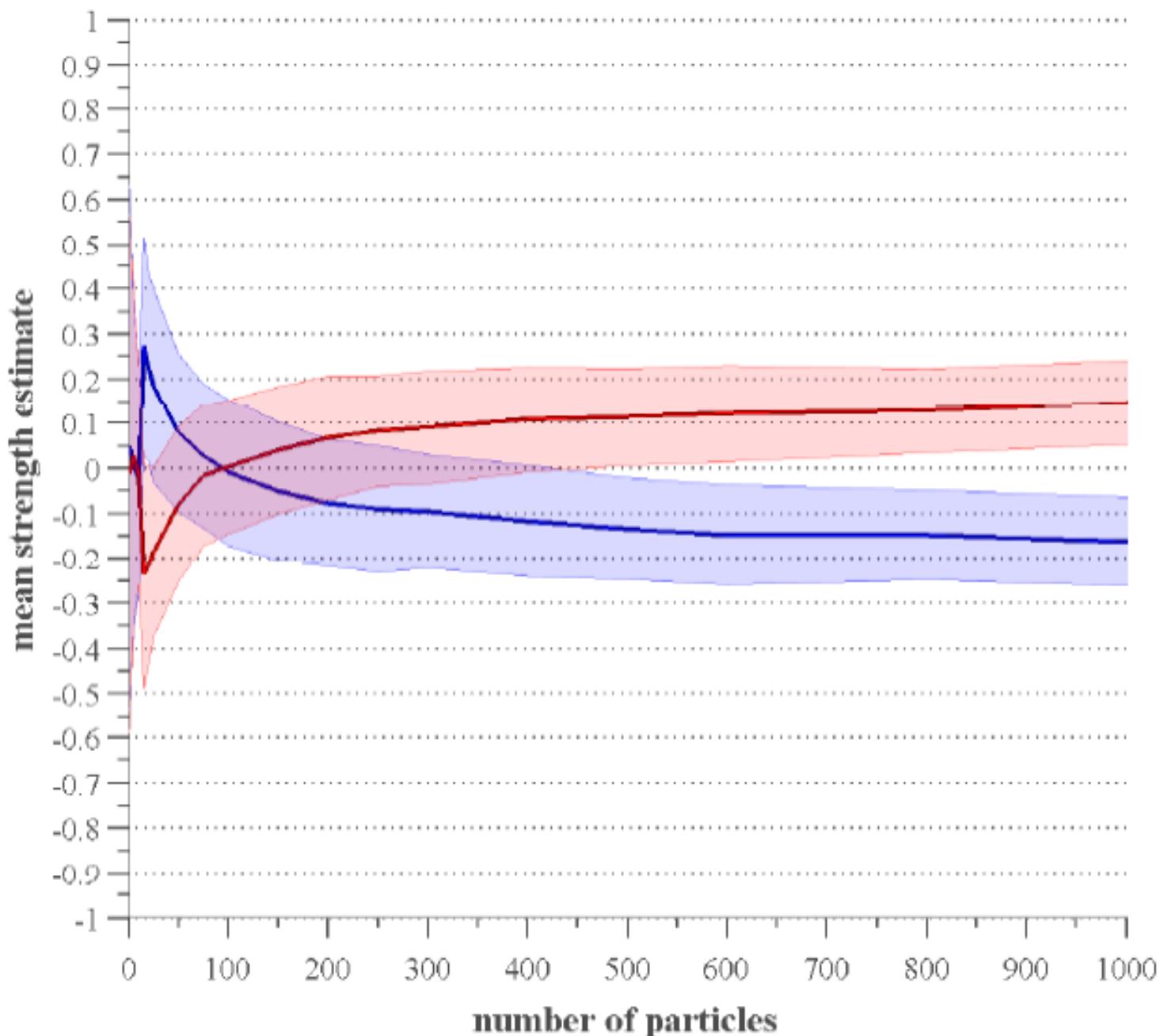


# Model 3: ESS with Rejuvenation

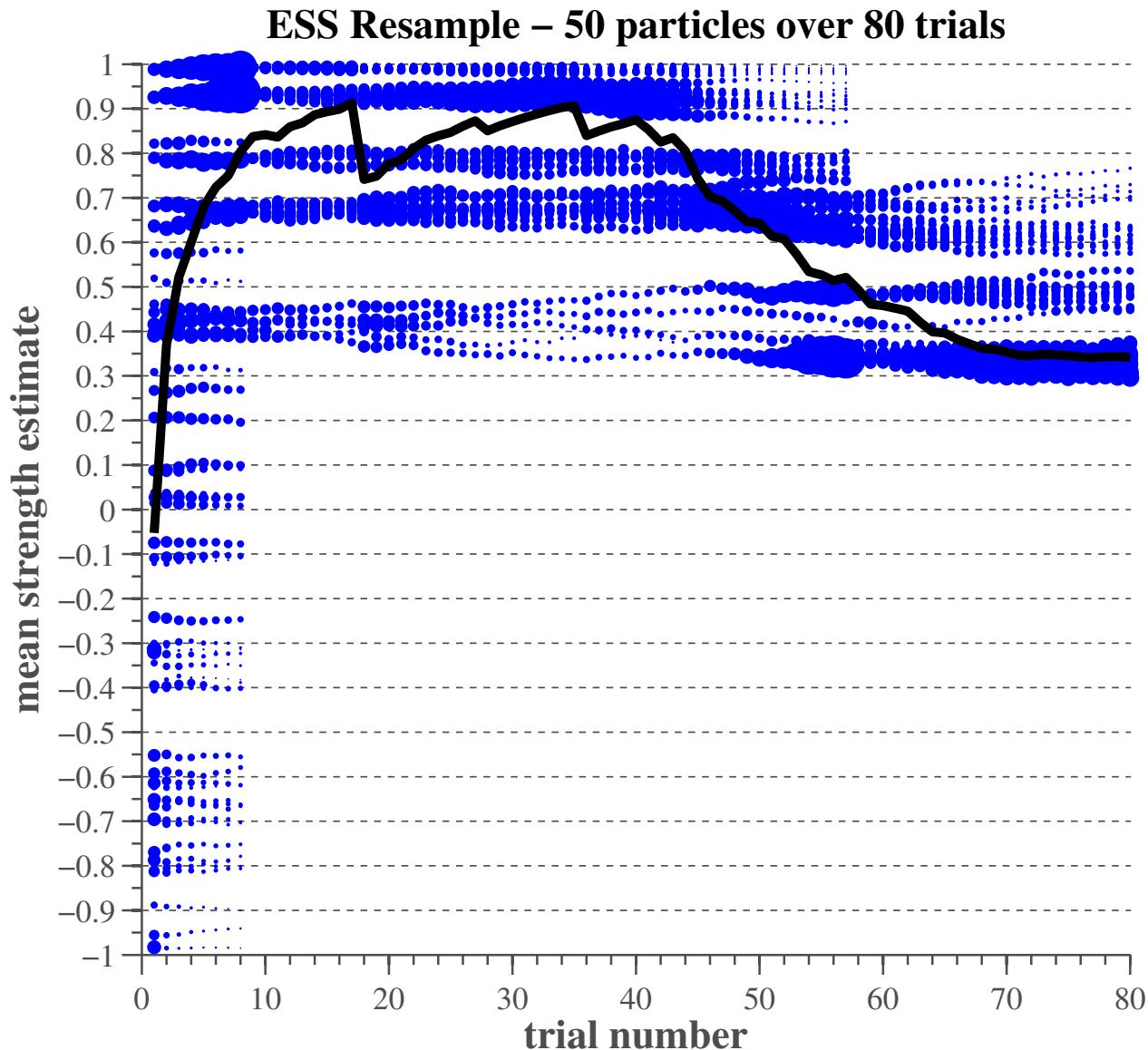
Resample using same ESS threshold, however,  
after resampling we perform Metropolis-  
Hastings

## ESS Resample with Rejuvenation

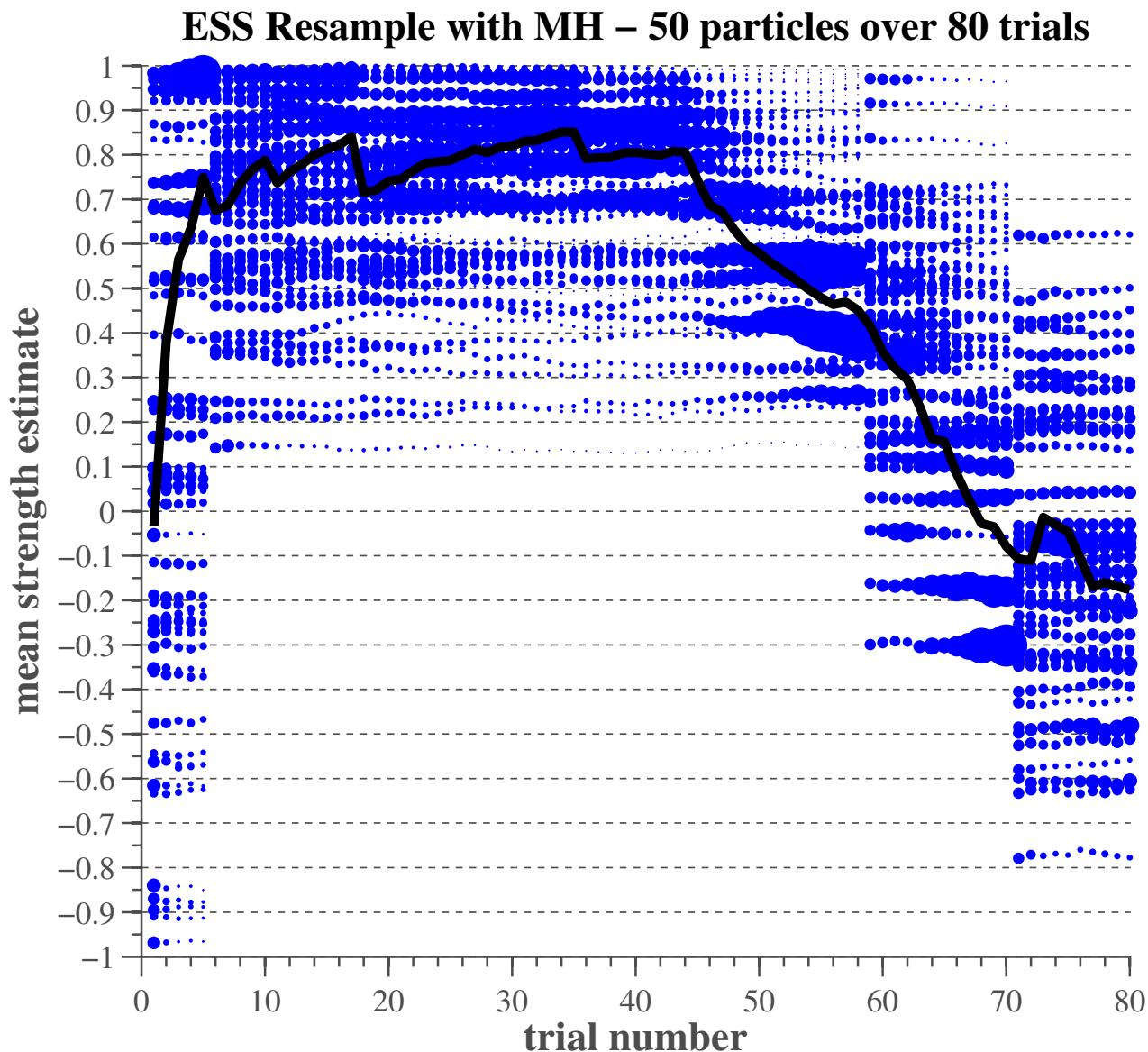
- generative  
block first
- preventative  
block first



# Closer look at rejuvenation



# Closer look at rejuvenation

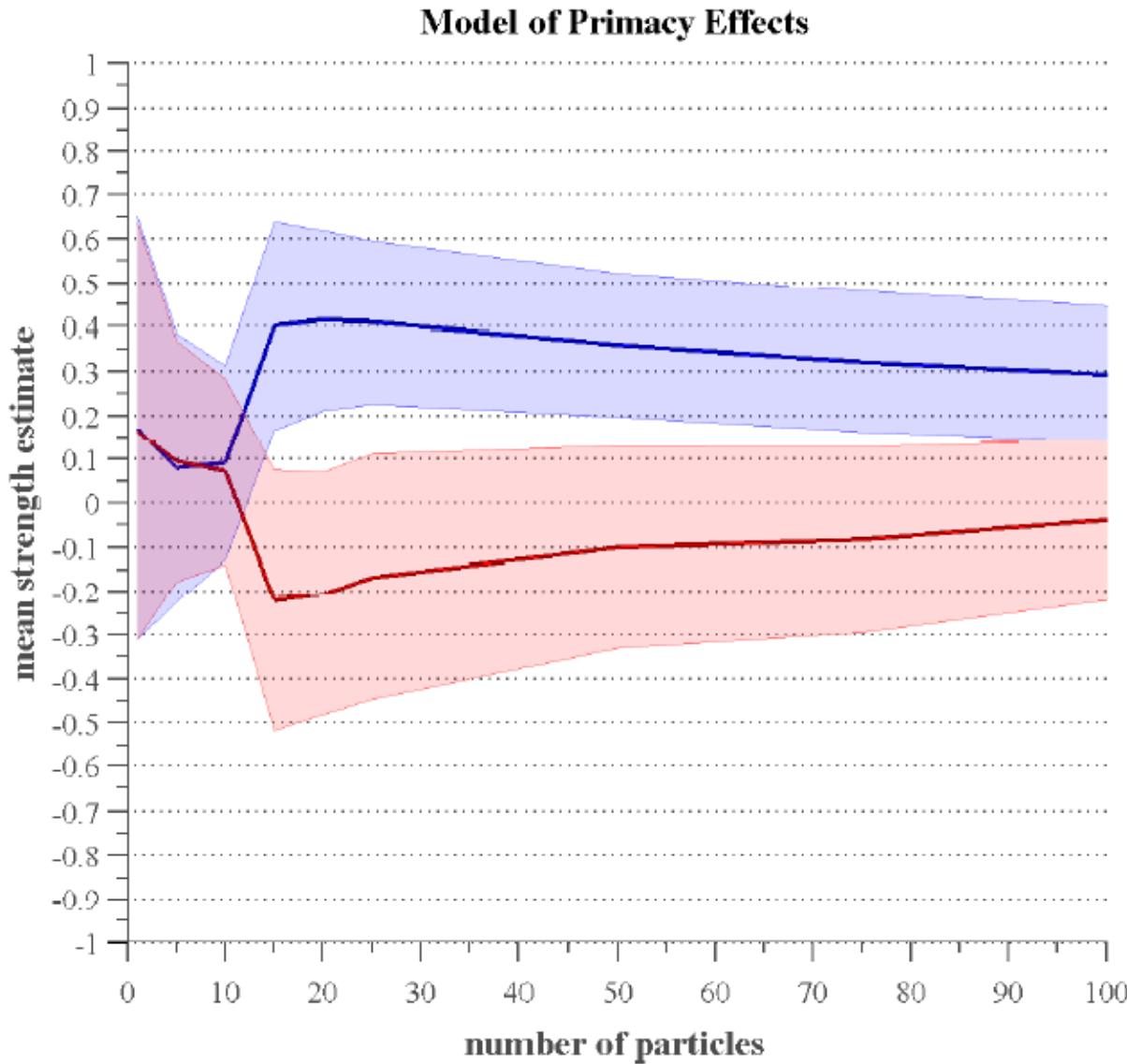


# Modeling human data

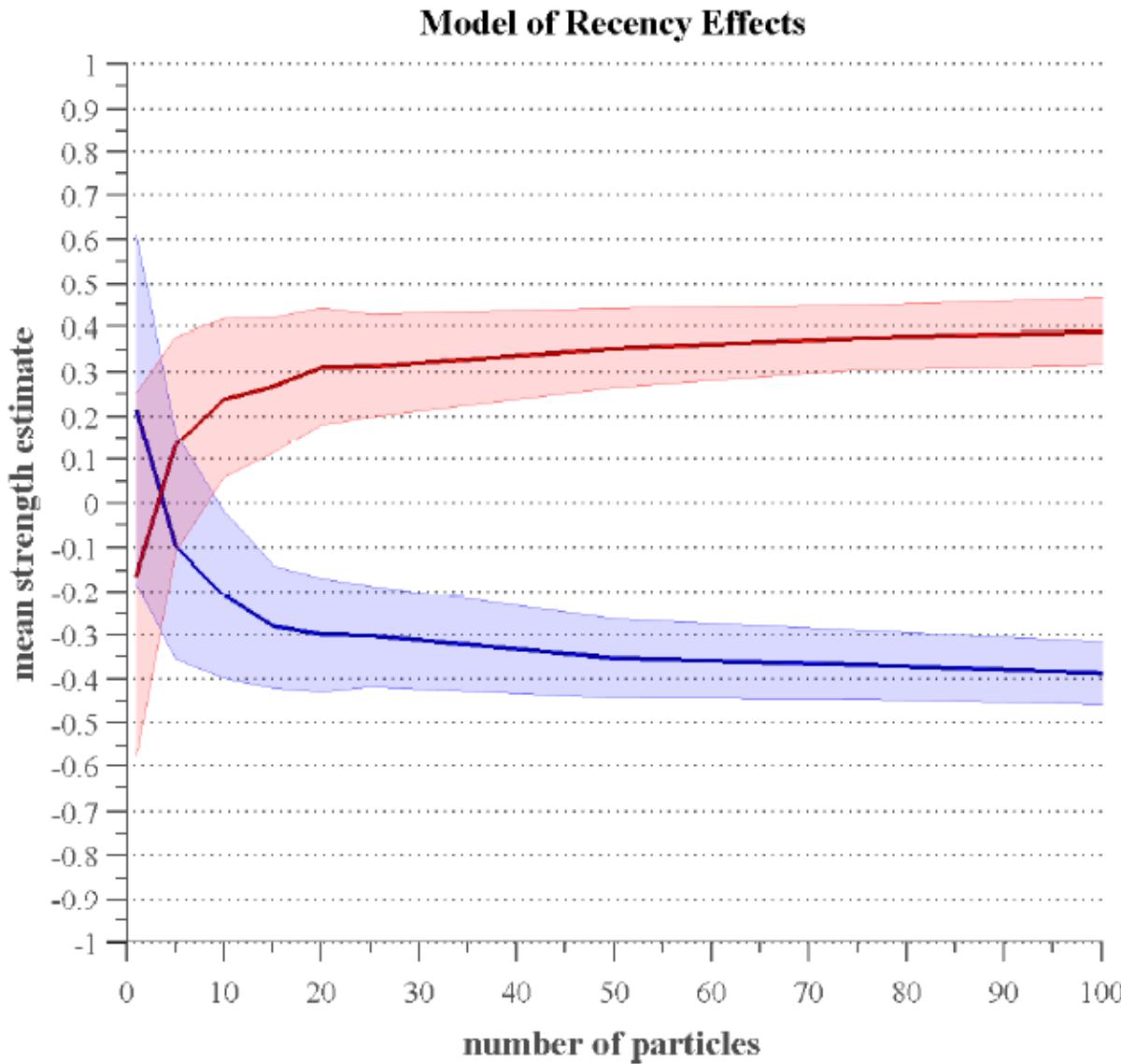
Frequency of judgment difference in the experimental setup

- only at end of trial – primacy
- every 10 trials – recency

# Modeling human data



# Modeling human data



# Conclusions

- Different resampling methods in a particle filter can produce different order effects in a causal learning task
- Provides a more consistent explanation of observed order effects in behavioral data

# Conclusions

Two key elements interacting:

- Filtering – in which we observe one data point at a time
- Rejuvenation – in which we consider all previously observed data

# Questions?

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<http://cocosci.berkeley.edu/josh/>

# Bayesian model of causal learning

What is the probability of the observed data given the strength estimates?

data		noisy-OR	noisy-AND-NOT
C	E	$s_1 \geq 0$	$s_1 < 0$
1	1	$s_0 + s_1 - s_0 s_1$	$s_0(1+s_1)$
1	0	$1-(s_0+s_1-s_0 s_1)$	$1-[s_0(1+s_1)]$
0	1	$s_0$	$s_0$
0	0	$1-s_0$	$1-s_0$

$$\Delta P$$

$$\Delta P = P(E/C) - P(E/\sim C)$$